AFM Notes, Unit 4 – Piecewise Functions

4-1 Functions

Name__________________________

Date____________ Period ______

Function:

Domain:

Increasing:

Decreasing:

Range:

Interval Notation:

Parenthesis, brackets or a combination of both.

(   ): use when there are unknown values, not included

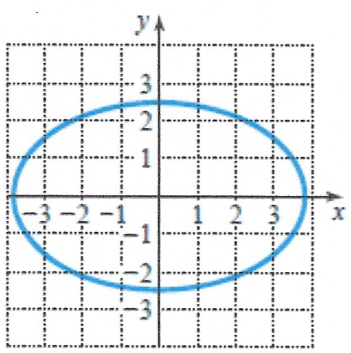
[   ]: values that you know the function can be equal to, included

Union "U":

The Vertical Line Test:

Examples: Determine if the graph is a function. If it is, state the domain and range.

1.



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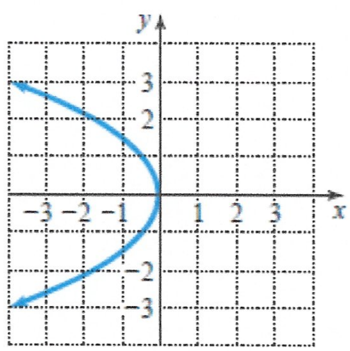

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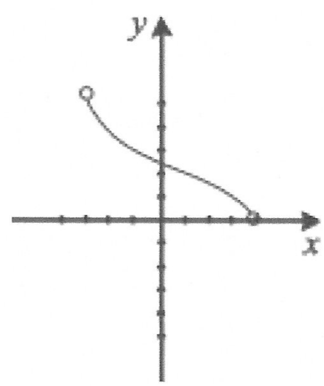

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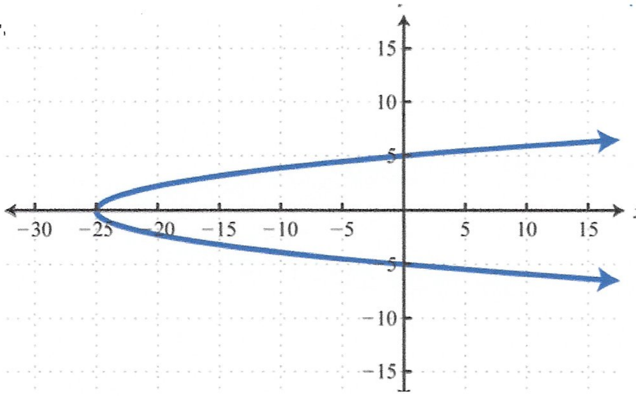

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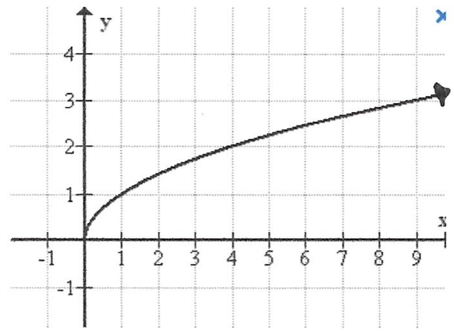


Graphing Functions:

Asymptotes:

Vertical:

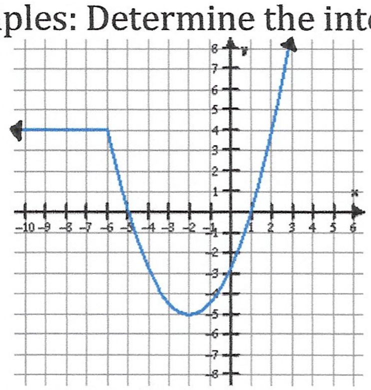
Horizontal:

Open Circle:

Closed Circle:

Examples: Determine the interval on which the function increases or decreases.

1.



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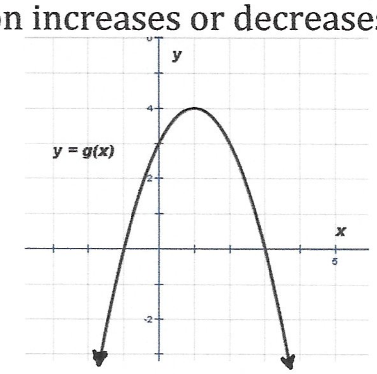

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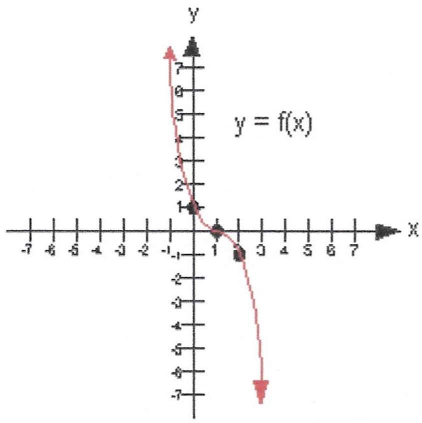

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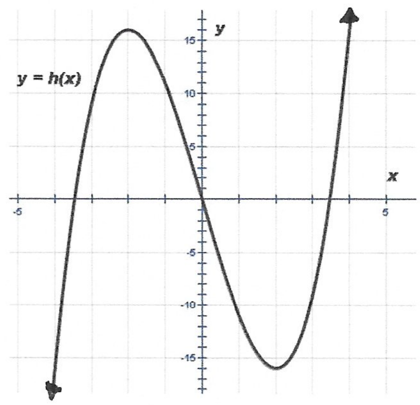
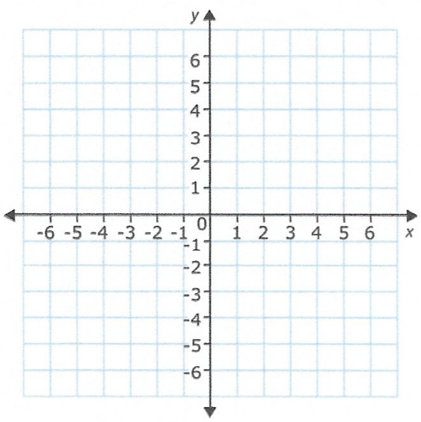
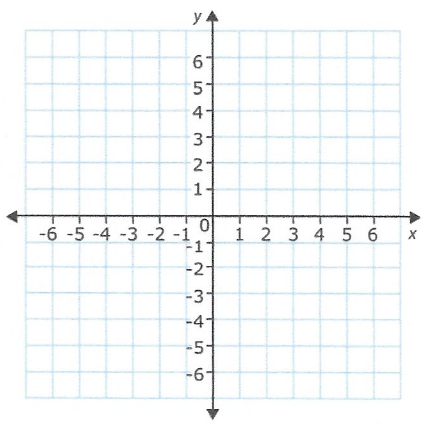


Sketch the graph of the following functions. Find the domain, range, and the interval of increase/decrease.

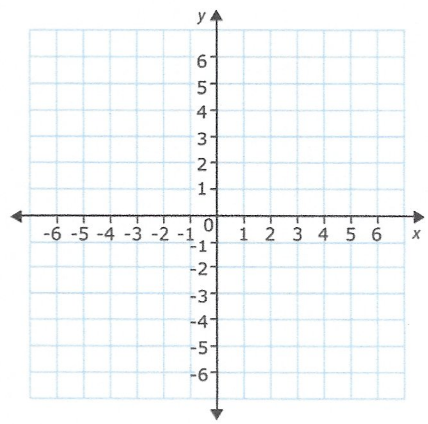
1. $f(x) = 1 - x$



2. $g(x) = x^2 - 3x$



3. $h(x) = \frac{4}{x-2}$



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Look for Restrictions:

Example: $f(x) = \frac{1}{x-4}$ When would this function be undefined?

Example: $f(x) = \sqrt{9 - x^2}$ When would this function be undefined?

Rational functions (fractions) have restrictions ______________________________________

____.

How can we find the restrictions?

Square Root Functions are undefined when ______________________________________

____.

How can we find the restrictions?

Examples:

1. $f(x) = \frac{1}{x^2 - x}$

2. $f(x) = \sqrt{9 - x^2}$

3. $h(t) = \frac{t}{\sqrt{t+1}}$

4. $f(x) = \frac{\sqrt{x+1}}{x-4}$

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Remember, a variable is just a place holder. So to evaluate a function f at a number, you substitute the number for the placeholder.

Let $f(x) = 3x^2 + x - 5$. Evaluate the function value.

(a) $f(-2)$

(c) $f(4)$

(b) $f(0)$

(d) $f\left(\frac{1}{2}\right)$

Evaluating a function with new variables will require a lot of simplification.
If $f(x) = 2x^2 + 3x - 1$, evaluate the following.

(a) $f(a)$

(b) $f(-a)$

(c) $f(a + h)$

(d) $\frac{f(a+h)-f(a)}{h}, h \neq 0$

Evaluate the Following Functions:

1. $w(n) = 4n + 2; Find\ w(3n)$

2. $p(t) = 4t - 5; Find\ p(t - 2)$

3. $w(a) = a + 3; Find\ w(a + 4)$

4. $h(x) = 4x - 2; Find\ h(x + 2)$

5. $g(n) = n^3 - 5n^2; Find\ g(-4n)$

6. $f(b) = b^2 - 2b; Find\ f(b^2)$

7. $p(a) = a^3 - 5; Find\ p(x - 4)$

AFM Notes, Unit 4 – Piecewise Functions
4-4 Parent Functions

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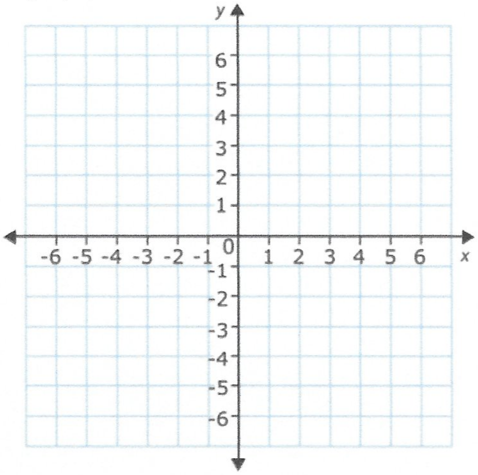
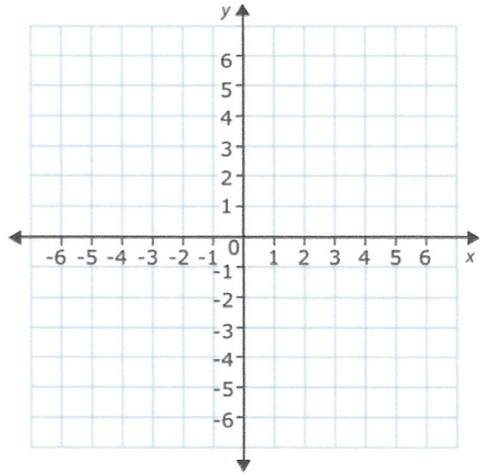
Graph each of the following functions.

1. $f(x) = x^2$



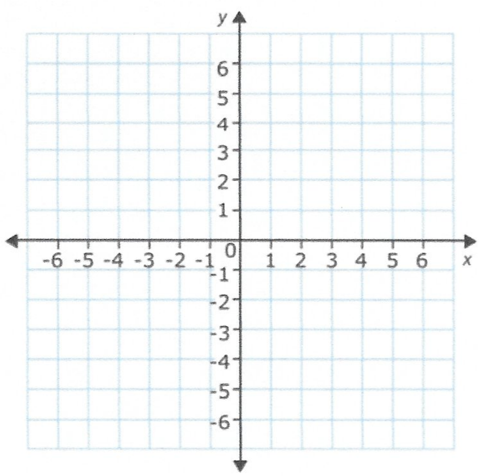
x-intercept: ________________
y-intercept: ________________
min/max point: ____________
Domain: ____________________
Range: ______________________
Type of Function:
____

2. $f(x) = x^3$



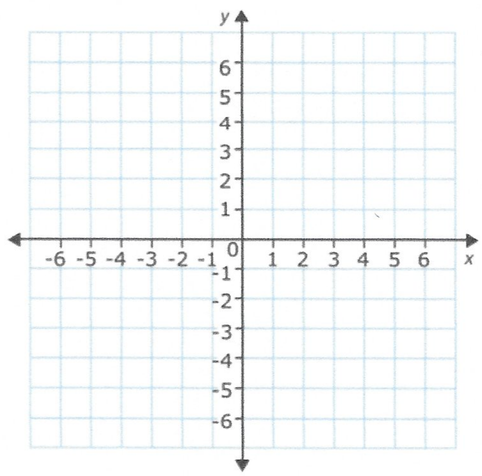
x-intercept: ________________
y-intercept: ________________
min/max point: ____________
Domain: ____________________
Range: ______________________
Type of Function:
____

3. $f(x) = \sqrt{x}$



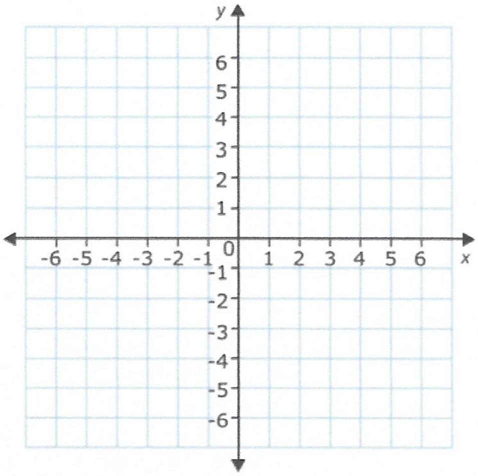
x-intercept: ________________
y-intercept: ________________
min/max point: ____________
Domain: ____________________
Range: ______________________
Type of Function:
____

4. $f(x) = |x|$



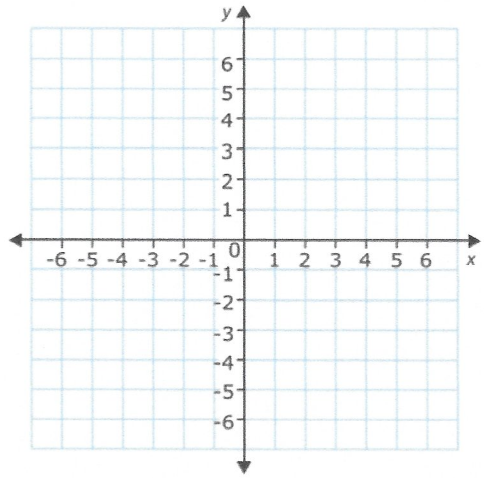
x-intercept: ________________
y-intercept: ________________
min/max point: ____________
Domain: ____________________
Range: ______________________
Type of Function:
____

5. $f(x) = x$



x-intercept: ____________________
y-intercept: ____________________
min/max point: ______________
Domain: ______________________
Range: ________________________
Type of Function:
____

6. $f(x) = 2^x$



x-intercept: ____________________
y-intercept: ____________________
min/max point: ______________
Domain: ______________________
Range: ________________________
Type of Function:
____

- These are called ______________ functions or __________ functions. There are sever other types but we will use these for our examples.
- We will investigate how we can ______________ the ______________ function to get a new graph.
- A transformation is a __________ in a graph from the parent function ____________ transformation ___________ the graph, but doesn't stretch or shrink the shape.

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Part 1: Graph the following on your calculator and write down the relationship between the toolkit/parent graph and its transformation. What do you do to the original to get the "new" graph?

Original #1: $y = x^2$

Original #2: $y = |x|$

a) $y = (x+1)^2$

b) $y = (x-4)^2$

c) $y = |x+2|$

d) $y = |x-5|$

e) In one complete sentence, predict what the graph of $y = (x+6)^3$ will look like.

f) In one complete sentence, predict what the graph of $y = \sqrt{x-3}$ will look like.

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x+c)$?

What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x-c)$?

Part 2: Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph?

Original #1: $y = x^2$

Original #2: $y = |x|$

a) $y = x^2 + 1$

b) $y = x^2 - 4$

c) $y = |x| + 2$

d) $y = |x| - 5$

e) In one complete sentence, predict what the graph of $y = x^3 + 6$ will look like.

f) In one complete sentence, predict what the graph of $y = \sqrt{x} - 3$ will look like.

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x) + c$?

What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x) - c$?

Part 3: Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph? Specifically, look at the 3 original points compared to those 3 "new" points.

Original #1: $y = x^2$ (0,0); (1, 1); (2, 4)

Original #2: $y = |x|$ (0,0); (-1, 1); (2, 2)

a) $y = 4x^2$

b) $y = \frac{1}{4}x^2$

c) $y = 2|x|$

d) $y = -|x|$

e) The points (0,0), (1,1) and (4,2) are on the graph of $y = \sqrt{x}$. What corresponding points do you think will be on the graph of $y = 4\sqrt{x}$?

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = kf(x)$ where k is positive?

What transformation occurs to the graph of $y = f(x)$ if you have $y = -f(x)$?

Part 4: Look at the graph of $y = \sqrt{x}$ and specifically the points (1,1), (4, 2) and (9, 3).

a) Graph $y = \sqrt{2x}$. Fill in the blanks in these ordered pairs: (1, ______); (4, ______); (9, ______);

b) Now graph $y = \sqrt{4x}$. Fill in the blanks in these ordered pairs: (1, ______); (4, ______); (9, ______);

c) Now graph $y = \sqrt{\frac{1}{2}x}$. Fill in the blanks in these ordered pairs: (1, ______); (4, ______); (9, ______);

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(kx)$ where k is positive?

Part 5: Look at the graph of $y = \sqrt{-x}$. Describe the difference between it and the graph of $y = \sqrt{x}$.

a) Compare $y = x^3$ and $y = (-x)^3$. Describe the difference. What happened to the original to get the new graph?

<u>**Generalize**</u>: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(-x)$?

Part 6: Put it all together! Without using your calculator, describe the transformations of the graphs. Without graphing, how will the following functions be changed from the parent graph?

a) $f(x) = -|x+1| + 1$

c) $q(x) = 2x^2 + 4$

b) $g(x) = -\sqrt{x+2} - 3$

d) $r(x) = 2 - \frac{1}{2}(x-3)^2$

Part 7: Think outside the box.

a) Predict what will happen to the graph of $y = \sqrt{-x+1}$. Do not use your calculator yet!

b) Now, using your calculator graph $y = \sqrt{-x+1}$. What happened to the graph? Were you correct?

c) Why do you think the rules applied differently to the graph?

Transformation Rules

Horizontal Transformations:

F(x+c) ______________________________

F(x-c) ______________________________

Vertical Transformations

F(x) + C ______________________________

F(x) – C ______________________________

Reflections

-F(X) ______________________________

F(-x) ______________________________

Stretch/Shrink

a*F(x) when $a > 1$ ______________________________

a*F(x) when $0 < a < 1$ ______________________________

F(a*x) when $a > 1$ ______________________________

F(a*x) when $0 < a < 1$ ______________________________

Transformations of Functions Practice

1. Given the function $f(x) = x^2$, write the function whose graph of f(x) is:

A. shifted 6 units to the left

B. reflected about the y-axis

C. reflected about the x-axis

D. shifted 5 units up

E. vertically stretched by a factor of 4

F. horizontally stretched (compressed) by a factor of 1/3

2. Given the function $f(x) = |x|$, write the function whose graph of f(x) is:

A. shifted 6 units to the left

B. reflected about the y-axis

C. reflected about the x-axis

D. shifted 5 units up

E. vertically stretched by a factor of 4

F. horizontally stretched (compressed) by a factor of 1/3

3. Write a function that is obtained after the following transformations are applied to y = |x|.

A. shift 2 units up, reflect about the x-axis then about the y-axis.

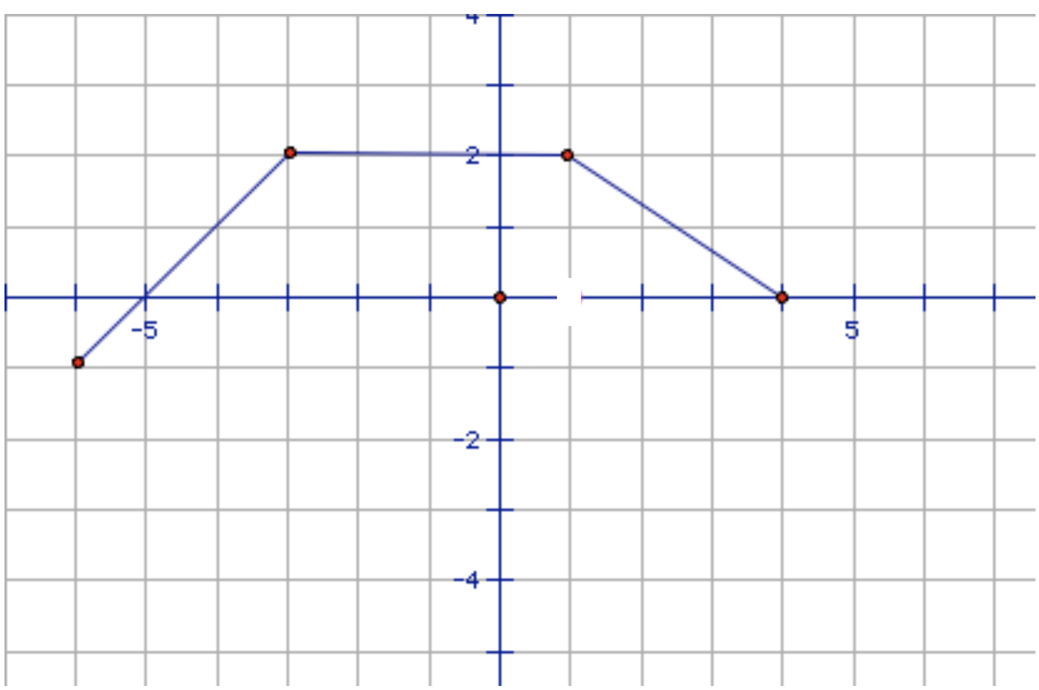
B. reflect over the x-axis, shift 3 units left and 2 units up.

4. Consider the following function f(x):

A. Graph f(x – 3)

B. Graph f(-x)

C. Graph –f(x)



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Piecewise Functions:

Continuous:

Discontinuous:

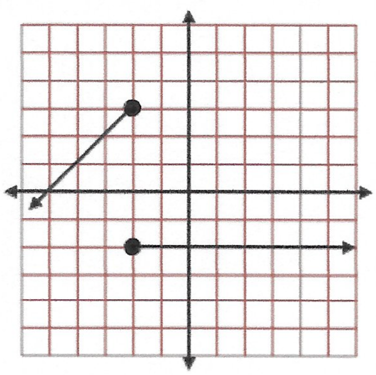
Evaluating Piecewise Functions

Evaluate the following function at $x = -2, 1, 2, and\ 3$

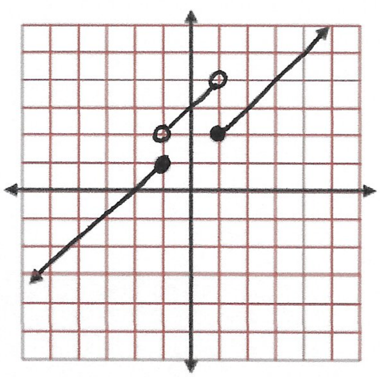
$$f(x) = \begin{cases} 1 - x, if\ x \le 1 \\ x^2, if\ x > 1 \end{cases}$$

Evaluate the following functions

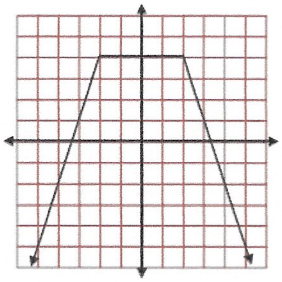
1. (a) Find the domain and range of the graph
   (b) Find the values for h(-2), h(0), h(-3)



2. (a) Find the domain and range of the graph
   (b) Find the values for h(-1), h(1), h(2)



3. (a) Find the domain and range of the graph
   (b) Find the values for h(-5), h(-2), h(2), h(4)



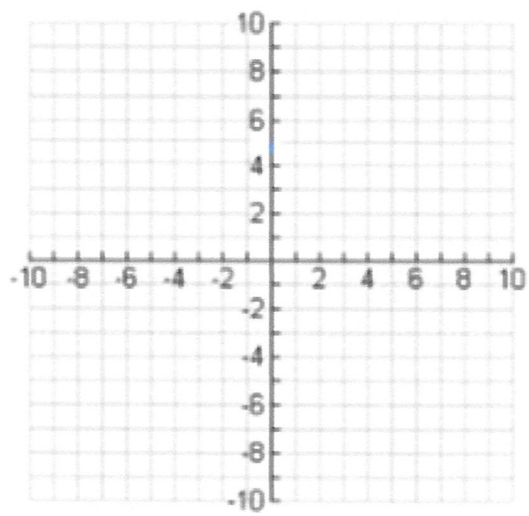
Graphing Piecewise Functions

Both of the following notations can be used to describe a piecewise function over the function's domain:

$$f(x) = \begin{cases} 2x & if[-5,2) \\ 5 & if[2,6] \end{cases} \quad \text{or} \quad f(x) = \begin{cases} 2x & ,-5 \le x < 2 \\ 5 & ,2 \le x \le 6 \end{cases}$$

4. Complete the following table of values for the piecewise function over the given domain.

x	f(x)
-5	
-3	
0	
1	
1.7	
1.9	
2	
2.2	
4	
6	



5. Graph the ordered pairs from your table to Sketch the graph of the piecewise function.

6. How many pieces does your graph have? Why?

7. Are the pieces rays or segments? Why?

8. Are all the endpoints solid dots or open dots or some of each? Why?

9. Were all these x values necessary to graph this piecewise function, or could this have been graphed using less points?

10. Which x values were "critical" to include in order to sketch the graph of this piecewise function?

11. Can you generalize which x-values are essential to input into your table to make a hand sketched graph of a piecewise linear function?

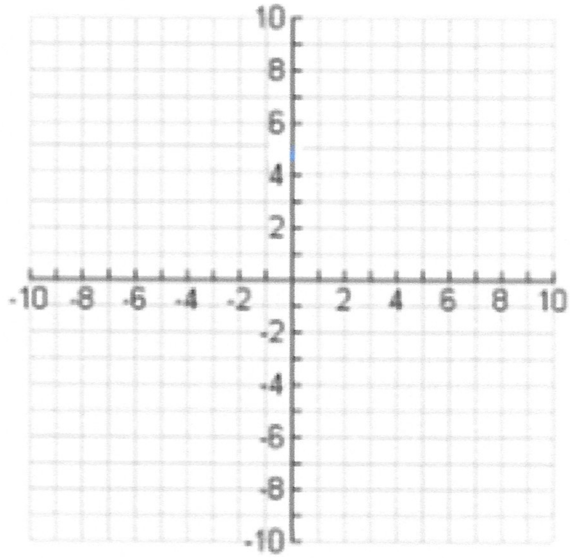
12. Now graph this piecewise function: $f(x) = \begin{cases} x+3 & ,-8 \le x < 1 \\ 10-2x & ,1 \le x \le 7 \end{cases}$

by completing a table of values for the piecewise function over the given domain.

x	f(x)

13. Why did you choose the x values you placed into the table?

14. Graph the ordered pairs from your table to Sketch the graph of the piecewise function.



15. How many pieces does your graph have? Why?

16. Are the pieces rays or segments? Why?

17. Are all the endpoints filled circles or open circles or some of each? Why?

18. Was it necessary to evaluate both pieces of the function for the x-value 1? Why or why not?

19. Which x values were "critical" to include in order to graph this piecewise function? Explain.

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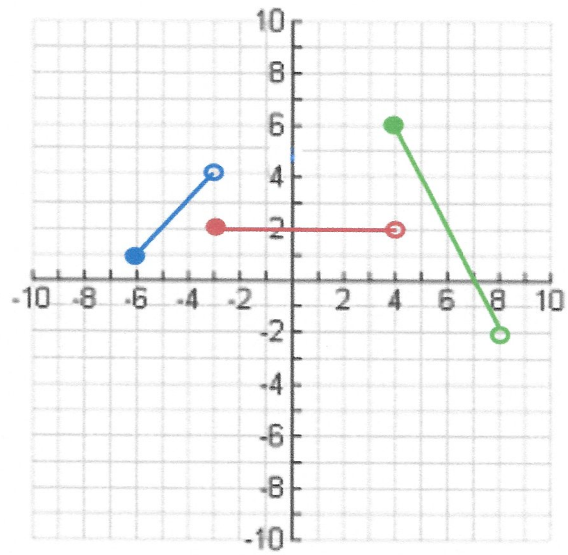
Writing piecewise functions given a graph.

1. Can you identify the equations of the lines that contain each segment?

   a. Left segment equation=

   b. Middle segment equation=

   c. Right segment equation=



2. Next, list the domain of each segment.

   a. Left segment domain=

   b. Middle segment domain=

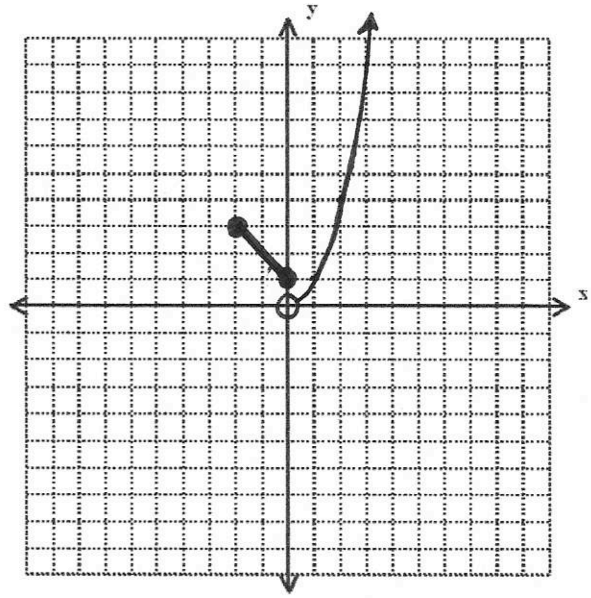
   c. Right segment domain=

3. Now, put the domain together with the equations to write the piecewise function for the graph.

$$f(x) = \begin{cases} \\ \\ \end{cases}$$

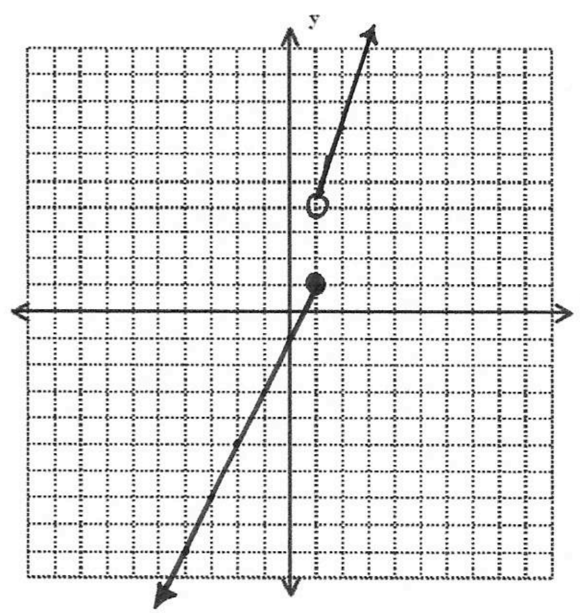
Write a piecewise function for the given graphs.
Example: 1

$f(x) = \{$



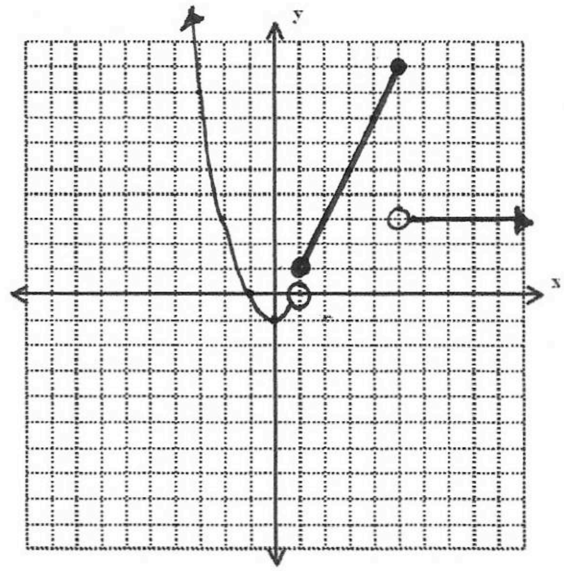
Example 2:

$f(x) = \{$



Example 3:

$$f(x) = \begin{cases} \\ \\ \end{cases}$$



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1. A long distance telephone charges 99 cents for any call up to 20 minutes in length and 7 cents for each additional minute. Use bracket notation to write a formula for the cost, C, of a call as a function of its length time, t, in minutes. Graph the function. How much does it cost to talk for 10 minutes? 25 minutes?

2. Suppose a carpet store sells carpet for $10 per square yard for the 100 sq yards purchased, and then lowers the price to $7 per square yard after the first 100 yards have been purchased. Find a function, $C = f(x)$, that gives the cost of purchasing any number of square yards of carpet between 0 and 200 square yards. How much does it cost for 50 square yards? 150 square yards?

3. A company charges $200 a month to organize a company's payroll for up to 20 employees and an additional $100 a month for each 20 employees over 20. Find a function, $P = f(x)$, that gives the payroll amount for 100 employees in one month. Graph the function.