AFM Notes, Unit 4 – Piecewise Functions
4-1 Functions

Name Key
Date____________ Period ______

Function: A rule that assigns each element, x, in a set "A" to EXACTLY ONE element, y, in a set "B".
x can only equal one y-value

Domain: All possible input values (x-values)

Increasing: As x increases, y increases

Decreasing: As x increases, y decreases

Range: All possible output values (y-values)

Interval Notation: Used when writing domain and range

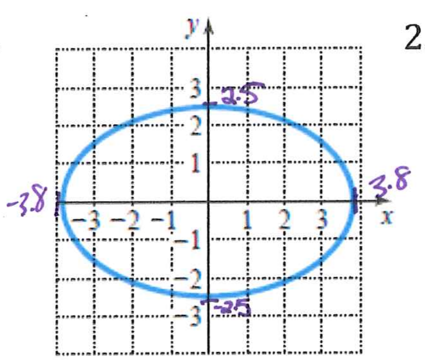
Parenthesis, brackets or a combination of both.
(   ): use when there are unknown values, not included
[   ]: values that you know the function can be equal to, included

Union "U": Combines multiple sets

The Vertical Line Test: If a vertical line intersects the graph more than once, then the graph is not a function.

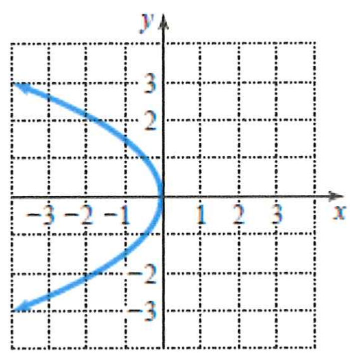
Examples: Determine if the graph is a function. If it is, state the domain and range.

1.



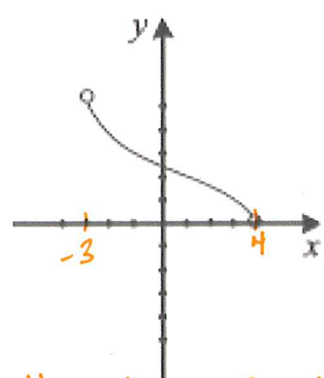
Not a Function
(Fails VLT)
Domain: [-3.8, 3.8]
Range: [-2.5, 2.5]

2.



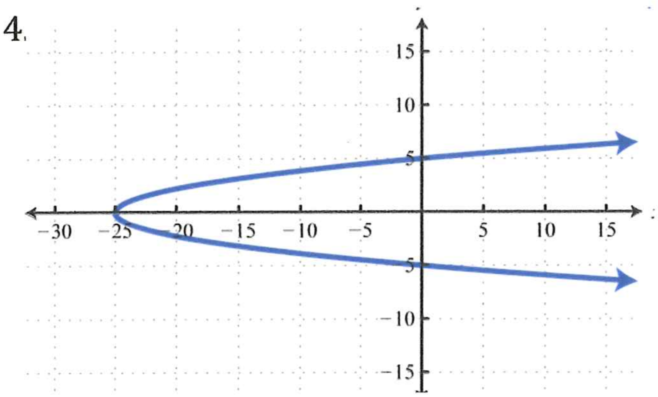
Not a Function
(Fails VLT)
Domain: (-∞, 0]
Range: (-∞, ∞)

3.



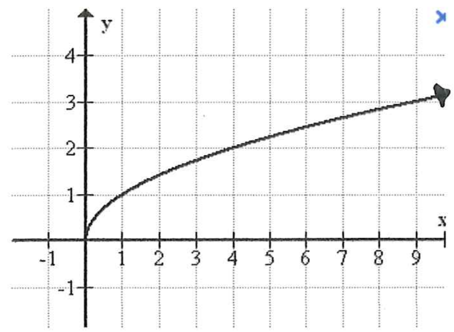
Yes it's a Function
Domain: (-3, 4)
Range: (0, 5)

4.



Not a Function
Domain: [-25, ∞)
Range: (−∞, ∞) All Real #s

5.



Yes it's a Function
Domain: [0, ∞)
Range: [0, ∞)

Graphing Functions:

Asymptotes: A boundary line (imaginary) that a graph gets infinitly close to but never crosses.

Vertical: X = a number, affects the domain

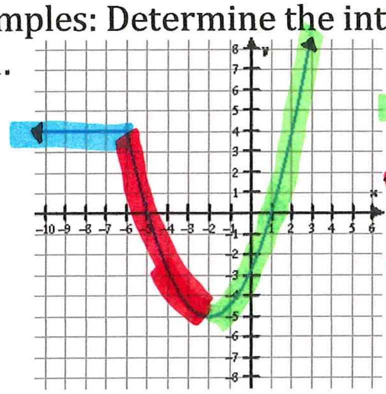
Horizontal: y = a number, affects the range

Open Circle: O means the value is NOT included

Closed Circle: ● means the value IS INCLUDED

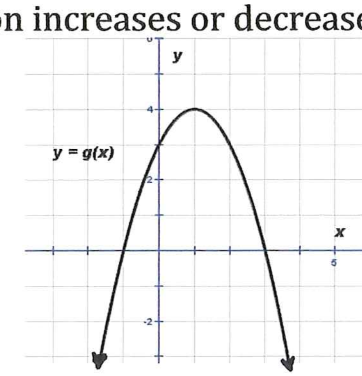
Examples: Determine the interval on which the function increases or decreases.

1.



Increases: (−∞, −6)
Decreases: (−6, −2)
Constant: (−2, ∞)

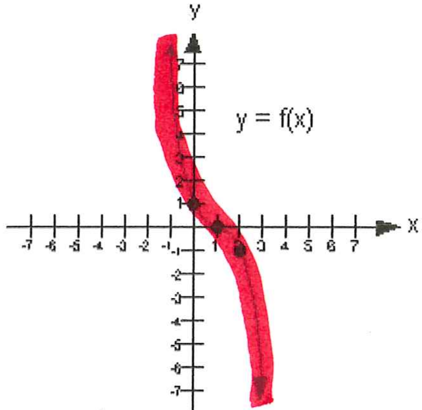
2.



Increases: (−∞, 1)
Decreases: (1, ∞)
Constant: Never

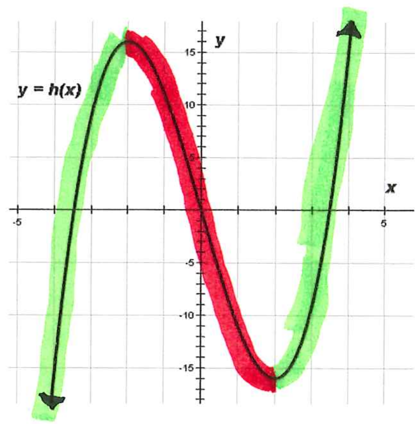
SN: Always use parenthesis ( ) when giving the interval for when a graph is constant, increasing, and decreasing.

3.



Increasing: Never
Decreasing: $(-\infty, \infty)$
Constant: Never

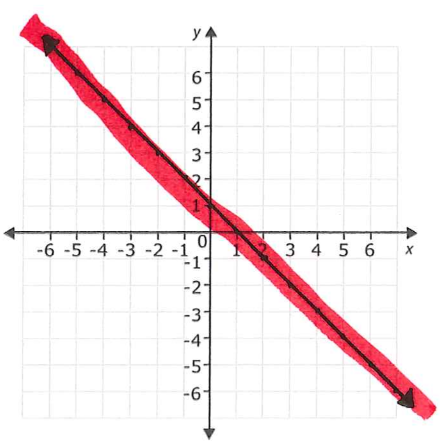
4.



Increasing: $(-\infty, -2) \cup (2, \infty)$
Decreasing: $(-2, 2)$
Constant: Never

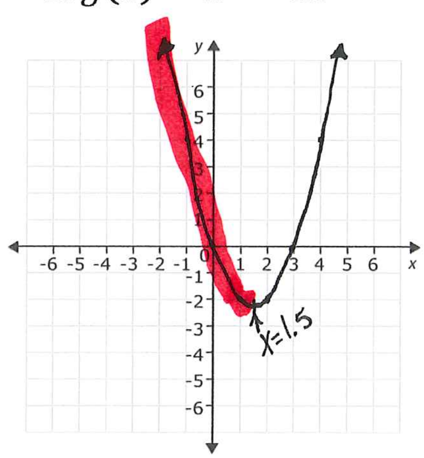
Sketch the graph of the following functions. Find the domain, range, and the interval of increase/decrease.

1. $f(x) = 1 - x$



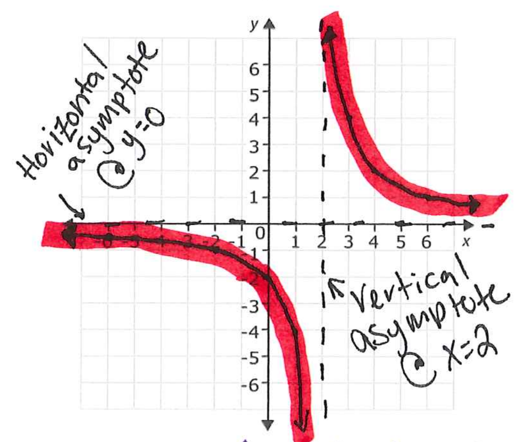
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Increasing: Never
Decreasing: $(-\infty, \infty)$

2. $g(x) = x^2 - 3x$



Domain: $(-\infty, \infty)$
Range: $[-2.25, \infty)$
Increasing: $(-\infty, 1.5)$
Decreasing: $(1.5, \infty)$

3. $h(x) = \frac{4}{x-2}$



Domain: $(-\infty, 2) \cup (2, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
Increasing: $(-\infty, 2) \cup (2, \infty)$
Decreasing: Never

AFM Notes, Unit 4 – Piecewise Functions
4-2 Finding Domain without a Graph

Name Key
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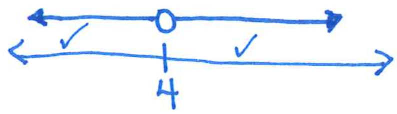
Look for Restrictions:

Example: $f(x) = \frac{1}{x-4}$ When would this function be undefined?

$x - 4 \neq 0$
$+4 \quad +4$
$x \neq 4$

Undefined when the denominator is 0.

Domain: $(-\infty, 4) \cup (4, \infty)$

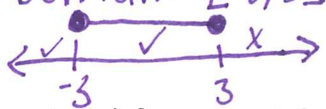


Example: $f(x) = \sqrt{9 - x^2}$ When would this function be undefined?

$9 - x^2 = 0$
$-x^2 = -9$
$\sqrt{x^2} = \sqrt{9}$
$x = \pm 3$

Undefind when the radicand is negative

Domain: $[-3, 3]$



Rational functions (fractions) have restrictions when the denominator is equal to zero.

How can we find the restrictions?

Set the denominator equal to zero then solve for x.

Square Root Functions are undefined when radicand is less than zero. (negative)

How can we find the restrictions?

Set the radicand equal to zero and solve for x.

Examples:

1. $f(x) = \frac{1}{x^2-x}$

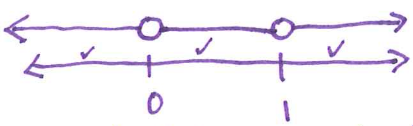
$x^2 - x \neq 0$ set denominator equal to 0

$x(x-1) \neq 0$ factor

$x \neq 0$ $\quad x-1=0$ zero property

$\quad +1 \ +1$

$x \neq 1$



Domain: $(-\infty,0)\cup(0,1)\cup(1,\infty)$

2. $f(x) = \sqrt{9-x^2}$

$9-x^2 \not< 0$ Radicand can't be less than zero

$(3-x)(3+x) \not< 0$ Factor

$3-x=0$ $\quad 3+x=0$ Zero Property

$-3 \quad -3$ $\quad -3 \quad -3$

$-x=-3$ $\quad x=-3$

$x=3$

Domain: $[-3,3]$

3. $h(t) = \frac{t}{\sqrt{t+1}}$

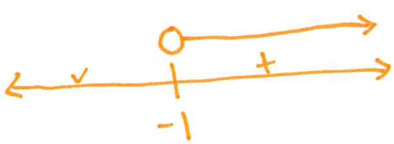
Can't divide by zero.

Radicand can't be less than zero.

$t+1 \neq 0$ $\quad t+1 \not< 0$

$-1 \ -1$ $\quad -1 \ -1$

$t=-1$ $\quad t \not< -1$



Domain: $(-1,\infty)$

4. $f(x) = \frac{\sqrt{x+1}}{x-4}$

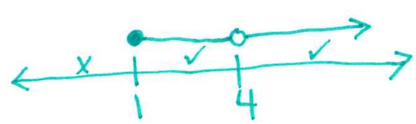
Can't divide by zero.

Radicand can't be less than zero.

$x-4=0$ $\quad x+1 \not< 0$

$+4 \ +4$ $\quad -1 \ -1$

$x=4$ $\quad x \not< 1$



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Remember, a variable is just a place holder. So to evaluate a function f at a number, you substitute the number for the placeholder.

Let $f(x) = 3x^2 + x - 5$. Evaluate the function value.

(a) $f(-2) = 3(-2)^2 + (-2) - 5$
$= 3(4) - 2 - 5$
$= 12 - 2 - 5$
$= 5$

(b) $f(0) = 3(0)^2 + 0 - 5$
$= 0 + 0 - 5$
$= -5$

(c) $f(4) = 3(4)^2 + 4 - 5$
$= 3(16) + 4 - 5$
$= 48 + 4 - 5$
$= 47$

(d) $f\left(\frac{1}{2}\right) = 3(1/2)^2 + 1/2 - 5$
$= 3(1/4) + 1/2 - 5$
$= \frac{3}{4} + \frac{1}{2} - 5$
$= -3.75$ or $-3\frac{3}{4}$

Evaluating a function with new variables will require a lot of simplification.
If $f(x) = 2x^2 + 3x - 1$, evaluate the following.

(a) $f(a) = 2(a)^2 + 3(a) - 1$
$= 2a^2 + 3a - 1$

(b) $f(-a) = 2(-a)^2 + 3(-a) - 1$
$= 2a^2 - 3a - 1$

(c) $f(a+h) = 2(a+h)^2 + 3(a+h) - 1$
$= 2(a+h)(a+h) + 3a + 3h - 1$
$= 2[a^2 + ah + ah + h^2] + 3a + 3h - 1$
$= 2a^2 + 2ah + 2ah + 2h^2 + 3a + 3h - 1$
$= 2a^2 + 4ah + 2h^2 + 3a + 3h - 1$

(d) $\frac{f(a+h) - f(a)}{h}, h \neq 0$

$$\frac{(2a^2 + 4ah + 2h^2 + 3a + 3h - 1) - (2a^2 + 3a - 1)}{h}$$

$$\frac{\cancel{2a^2} + 4ah + 2h^2 + \cancel{3a} + 3h \cancel{-1} \cancel{-2a^2} \cancel{-3a} \cancel{+1}}{h}$$

$$\frac{4ah + 2h^2 + 3h}{h} = 4a + 2h + 3$$

Evaluate the Following Functions:

1. $w(n) = 4n + 2; Find\ w(3n)$

   $w(3n) = 4(3n) + 2$

   $w(3n) = 12n + 2$

2. $p(t) = 4t - 5; Find\ p(t - 2)$

   $p(t-2) = 4(t-2) - 5$

   $p(t-2) = 4t - 8 - 5$

   $p(t-2) = 4t - 13$

3. $w(a) = a + 3; Find\ w(a + 4)$

   $w(a+4) = (a+4) + 3$

   $w(a+4) = a + 4 + 3$

   $w(a+4) = a + 7$

4. $h(x) = 4x - 2; Find\ h(x + 2)$

   $h(x+2) = 4(x+2) - 2$

   $h(x+2) = 4x + 8 - 2$

   $h(x+2) = 4x + 6$

5. $g(n) = n^3 - 5n^2; Find\ g(-4n)$

   $g(-4n) = (-4n)^3 - 5(-4n)^2$

   $g(-4n) = -64n^3 - 5(16n^2)$

   $g(-4n) = -64n^3 - 80n^2$

6. $f(b) = b^2 - 2b; Find\ f(b^2)$

   $f(b^2) = (b^2)^2 - 2(b^2)$

   $f(b^2 = b^4 - 2b^2$

7. $p(a) = a^3 - 5; Find\ p(x - 4)$

   $p(x-4) = (x-4)^3 - 5$

   $p(x-4| = (x-4)(x-4)(x-4) - 5$

   $p(x-4) = (x^2 - 8x + 16)(x-4) - 5$

   $p(x-4) = x^3 - 8x^2 + 16x - 4x^2 + 32x - 64 - 5$

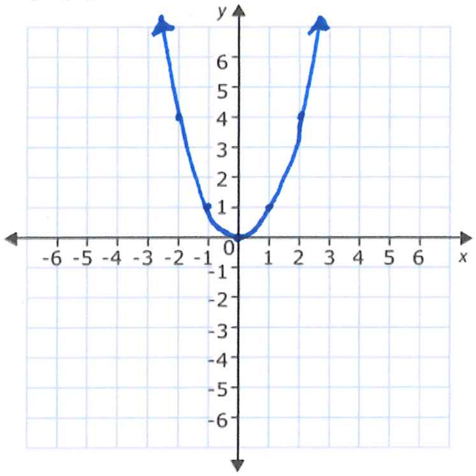
   $p(x-4) = x^3 - 12x^2 + 48x - 69$

AFM Notes, Unit 4 – Piecewise Functions
4-4 Parent Functions

Name key
Date_________ Period _____

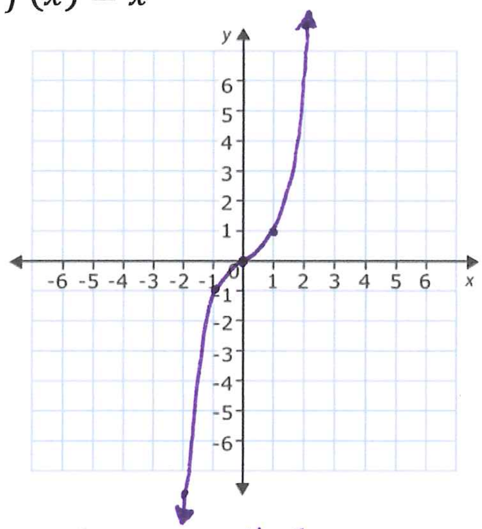
Graph each of the following functions.

1. $f(x) = x^2$



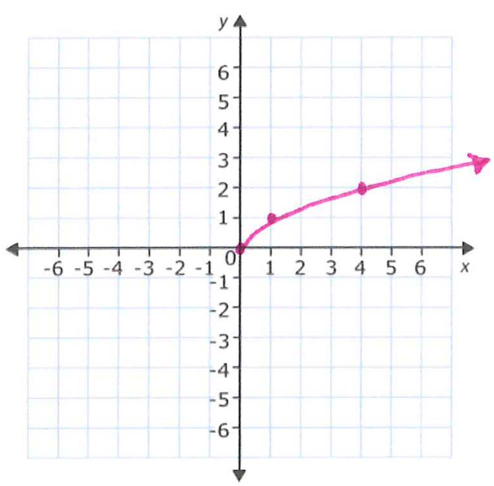
x-intercept: X=0
y-intercept: y=0
min/max point: (0,0) min
Domain: (-∞,∞)
Range: [0,∞)
Type of Function:
Quadratic

2. $f(x) = x^3$



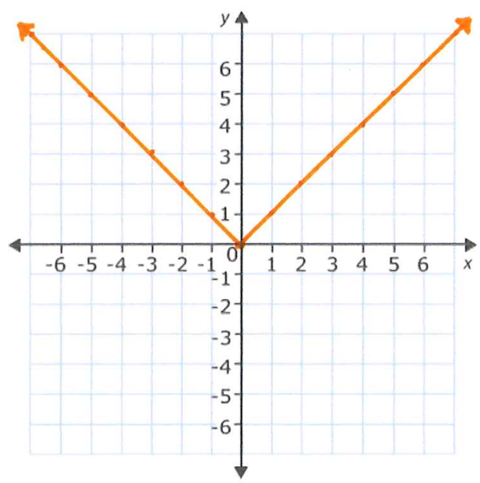
x-intercept: X=0
y-intercept: y=0
min/max point: None
Domain: (-∞,∞)
Range: (-∞,∞)
Type of Function:
Cubic

3. $f(x) = \sqrt{x}$



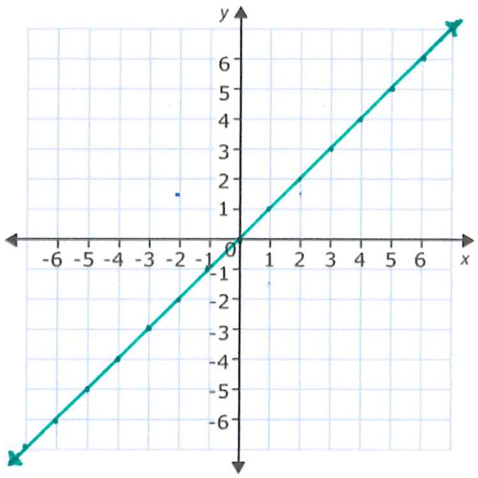
x-intercept: X=0
y-intercept: y=0
min/max point: (0,0) min
Domain: [0,∞)
Range: [0,∞)
Type of Function:
Square Root

4. $f(x) = |x|$



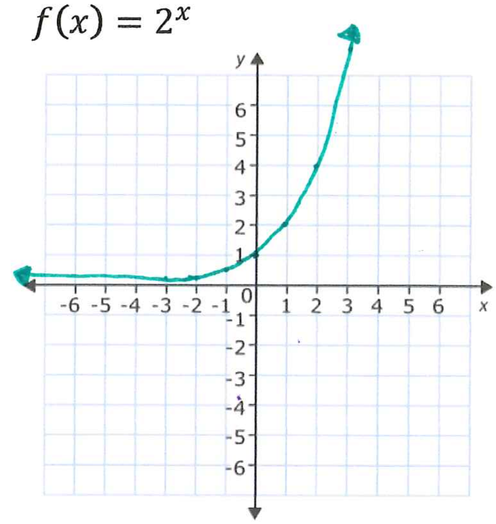
x-intercept: X=0
y-intercept: y=0
min/max point: (0,0) min
Domain: [0,∞)
Range: (-∞,∞)
Type of Function:
Absolute Value

5. $f(x) = x$



x-intercept: x=0
y-intercept: y=0
min/max point: None
Domain: (-∞,∞)
Range: (-∞,∞)
Type of Function: Linear

6. $f(x) = 2^x$



x-intercept: X=1
y-intercept: None
min/max point: None
Domain: (-∞,∞)
Range: (0,∞
Type of Function: Exponential

- These are called tool kit functions or parent functions. There are sever other types but we will use these for our examples.
- We will investigate how we can Shift the parent function to get a new graph.
- A transformation is a shift in a graph from the parent function isometric transformation Shift the graph, but doesn't stretch or shrink the shape.

AFM Notes, Unit 4 – Piecewise Functions
4-5 Transformations

Name Key
Date________ Period ______

Part 1: Graph the following on your calculator and write down the relationship between the toolkit/parent graph and its transformation. What do you do to the original to get the "new" graph?

Original #1: $y = x^2$

Original #2: $y = |x|$

a) $y = (x+1)^2$

Shift Left 1 unit

b) $y = (x-4)^2$

Shift Right 4 units

c) $y = |x+2|$

Shift Left 2 Units

d) $y = |x-5|$

Shift Right 5 units

e) In one complete sentence, predict what the graph of $y = (x+6)^3$ will look like.

$f(x) = x^3$ is the parent function and it will shift left 6 units.

f) In one complete sentence, predict what the graph of $y = \sqrt{x-3}$ will look like.

$f(x) = \sqrt{x}$ is the parent function and it will shift right 3 units.

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x+c)$?

Left C Units

What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x-c)$?

Right C Units

Part 2: Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph?

Original #1: $y = x^2$

Original #2: $y = |x|$

a) $y = x^2 + 1$

Up 1 Unit

b) $y = x^2 - 4$

Down 4 Units

c) $y = |x| + 2$

Up 2 Units

d) $y = |x| - 5$

Down 5 Units

e) In one complete sentence, predict what the graph of $y = x^3 + 6$ will look like.

$f(x) = x^3$ is the parent function and it will shift up 6 units.

f) In one complete sentence, predict what the graph of $y = \sqrt{x} - 3$ will look like.

$f(x) = \sqrt{x}$ is the parent function and it will shift down 2 units.

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x) + c$?

Up C Units

What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x) - c$?

Down C Units

Part 3: Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph? Specifically, look at the 3 original points compared to those 3 "new" points.

Original #1: $y = x^2$ (0,0); (1, 1); (2, 4)

Original #2: $y = |x|$ (0,0); (-1, 1); (2, 2)

a) $y = 4x^2$
(0,0)
(1,4)
(2,16)
Multiply x by 4

b) $y = \frac{1}{4}x^2$
(0,0)
(1,1/4)
(2,1)
Multiply x by $\frac{1}{4}$

c) $y = 2|x|$
(0,0)
(-1,2)
(2,4)
Multiply x by 2

d) $y = -|x|$
(0,0)
(-1,-1)
(2,-2)
Multiply x by -1

e) The points (0,0), (1,1) and (4,2) are on the graph of $y = \sqrt{x}$. What corresponding points do you think will be on the graph of $y = 4\sqrt{x}$? (0,0) (1,4) (4,8)

Multiply x by 4

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = kf(x)$ where k is positive?

Vertical Stretch by K when K>1
Vertical Shrink by K 0<K<1

What transformation occurs to the graph of $y = f(x)$ if you have $y = -f(x)$?

Reflect over x-axis

Part 4: Look at the graph of $y = \sqrt{x}$ and specifically the points (1,1), (4, 2) and (9, 3).

a) Graph $y = \sqrt{2x}$. Fill in the blanks in these ordered pairs: (1, 1.4); (4, 2.8); (9, 4.2);

b) Now graph $y = \sqrt{4x}$. Fill in the blanks in these ordered pairs: (1, 2); (4, 4); (9, 6);

c) Now graph $y = \sqrt{\frac{1}{2}x}$. Fill in the blanks in these ordered pairs: (1, .71); (4, 1.4); (9, 2.12);

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(kx)$ where k is positive?

Horizontal shrink when K>1
Horizontal Stretch when 0<K<1

Part 5: Look at the graph of $y = \sqrt{-x}$. Describe the difference between it and the graph of $y = \sqrt{x}$.

Reflected over y-axis

a) Compare $y = x^3$ and $y = (-x)^3$. Describe the difference. What happened to the original to get the new graph?

Reflected over y-axis

__**Generalize**__: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(-x)$?

Reflection over y-axis

Part 6: Put it all together! Without using your calculator, describe the transformations of the graphs. Without graphing, how will the following functions be changed from the parent graph?

a) $f(x) = -|x + 1| + 1$

Reflected over x-axis
Left 1 Unit
Up 1 unit

b) $g(x) = -\sqrt{x + 2} - 3$

Reflected over x-axis
Left 2 Units
Down 3 Units

c) $q(x) = 2x^2 + 4$

Vertical Stretch by 2
Up 4 Units

d) $r(x) = 2 - \frac{1}{2}(x - 3)^2$

Reflected over x-axis
Vertical Shrink by 2
Right 3 Units
Up 2 Units

Part 7: Think outside the box.

a) Predict what will happen to the graph of $y = \sqrt{-x + 1}$. Do not use your calculator yet!

Reflected over y-axis, Right 1 Unit

b) Now, using your calculator graph $y = \sqrt{-x + 1}$. What happened to the graph? Were you correct?

Yes

c) Why do you think the rules applied differently to the graph?

Because of the Reflection

Transformation Rules

Horizontal Transformations:

F(x+c) Left C units

F(x-c) Right C units

Vertical Transformations

F(x) + C Up C units

F(x) – C Down C units

Reflections

-F(X) Reflect over x-axis → Change sign of y-values

F(-x) Reflect over y-axis → Change sign of x-values

Stretch/Shrink

a*F(x) when a > 1 Vertical Stretch by a

a*F(x) when 0 < a < 1 Vertical Shrink by a

F(a*x) when a > 1 Horizontal Shrink by a

F(a*x) when 0 < a < 1 Horizontal Stretch by a

Transformations of Functions Practice

1. Given the function f(x) = x^2, write the function whose graph of f(x) is:

A. shifted 6 units to the left

$f(x) = (x+6)^2$

B. reflected about the y-axis

$f(x) = (-x)^2$

C. reflected about the x-axis

$f(x) = -x^2$

D. shifted 5 units up

$f(x) = x^2 + 5$

E. vertically stretched by a factor of 4

$f(x) = 4x^2$

F. horizontally stretched (compressed) by a factor of 1/3

$f(x) = (\frac{1}{3}x)^2$

2. Given the function $f(x) = |x|$, write the function whose graph of f(x) is:

A. shifted 6 units to the left

$f(x) = |x+6|$

B. reflected about the y-axis

$f(x) = |-x|$

C. reflected about the x-axis

$f(x) = -|x|$

D. shifted 5 units up

$f(x) = |x| + 5$

E. vertically stretched by a factor of 4

$f(x) = 4|x|$

F. horizontally stretched (compressed) by a factor of 1/3

$f(x) = |\frac{1}{3}x|$

3. Write a function that is obtained after the following transformations are applied to y = |x|.

A. shift 2 units up, reflect about the x-axis then about the y-axis.

$y = -|-x| + 2$

B. reflect over the x-axis, shift 3 units left and 2 units up.

$y = -|x+3| + 2$

4. Consider the following function f(x):

A. Graph f(x – 3)

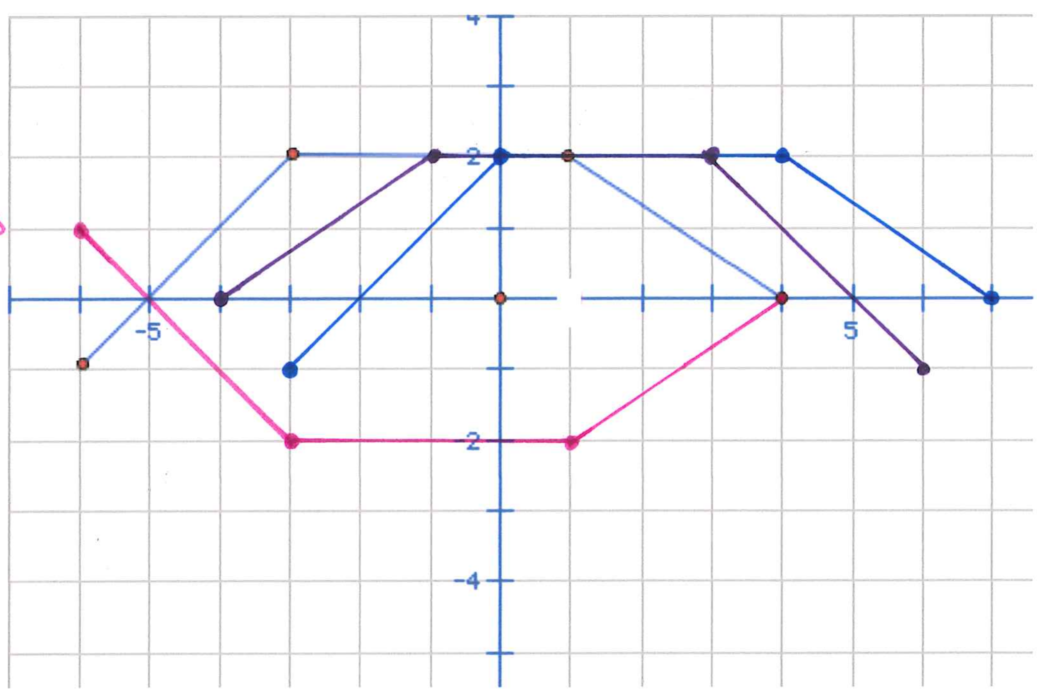
Right 3 Units

B. Graph f(-x)

Reflect over y-axis

C. Graph –f(x)

Reflect over x-axis



AFM Notes, Unit 4 – Piecewise Functions
4-6 Graphing Piecewise Functions

Name Key
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Piecewise Functions: A function defined by 2 or more equations with restrictions in the domain.

Continuous: Pieces connect to each other (You don't pick up your pencil)

Discontinuous: There are holes, jumps, or gaps (You have to pick up your pencil)

<u>Evaluating Piecewise Functions</u>

Evaluate the following function at $x = -2, 1, 2, and\ 3$

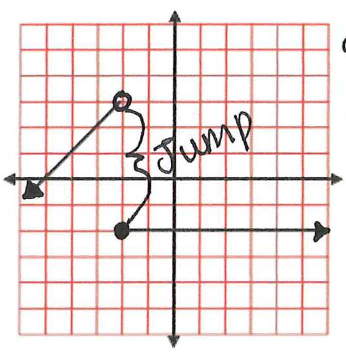
$$f(x) = \begin{cases} 1 - x, if\ x \le 1 \\ x^2, if\ x > 1 \end{cases}$$

$f(-2) = 1-(-2)$ $f(1) = 1-(1)$ $f(2) = (2)^2$ $f(3) = (3)^2$

$f(-2) = 3$ $f(1) = 0$ $f(2) = 4$ $f(3) = 9$

Evaluate the following functions

1. (a) Find the domain and range of the graph
   (b) Find the values for h(-2), h(0), h(-3)

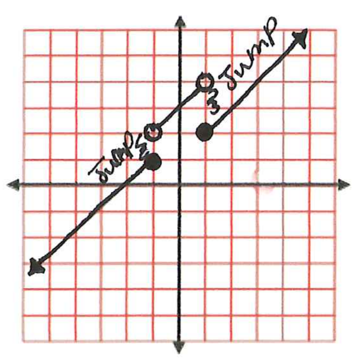


a) Domain: $(-\infty, \infty)$ Range: $(-\infty, 3)$

b) $h(-2) = -2$
$h(0) = -2$
$h(-3) = 2$

Discontinuous: Jump

2. (a) Find the domain and range of the graph
   (b) Find the values for h(-1), h(1), h(2)

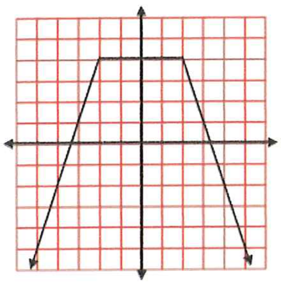


a) Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

b) $h(-1) = 1$
$h(1) = 2$
$h(2) = 3$

Discontinuous: 2 jumps

3. (a) Find the domain and range of the graph
   (b) Find the values for h(-5), h(-2), h(2), h(4)



a) Domain: $(-\infty, \infty)$ Range: $(-\infty, 4]$

b) $h(-5) = -5$
$h(-2) = 4$
$h(2) = 4$
$h(4) = 2$

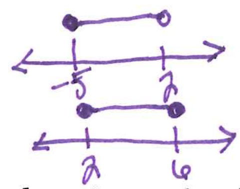
If $h(x) = -2$ then
$x = \underline{-4}$. $(-4, 4)$

Continuous

Graphing Piecewise Functions

Both of the following notations can be used to describe a piecewise function over the function's domain:

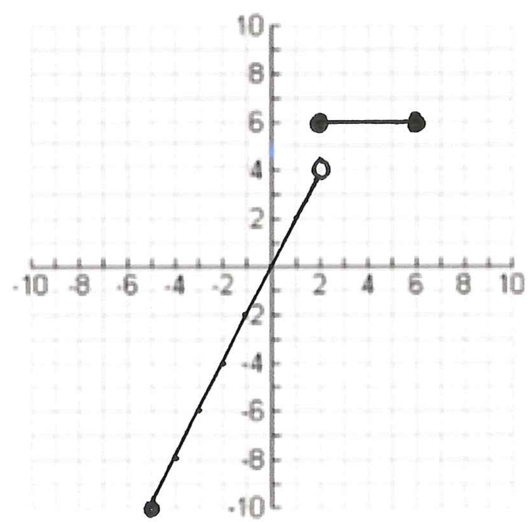
$$f(x) = \begin{cases} 2x & if[-5,2) \\ 5 & if[2,6] \end{cases} \quad \text{or} \quad f(x) = \begin{cases} 2x & ,-5 \le x < 2 \\ 5 & ,2 \le x \le 6 \end{cases}$$



Both Notations Can be used.

4. Complete the following table of values for the piecewise function over the given domain.

x	f(x)
-5	2(-5) = -10
-3	2(-3) = -6
0	2(0) = 0
1	2(1) = 2
1.7	2(1.7) = 3.4
1.9	2(1.9) = 3.8
2	5
2.2	5
4	5
6	5



5. Graph the ordered pairs from your table to Sketch the graph of the piecewise function.

6. How many pieces does your graph have? Why?

2, because there are 2 functions

7. Are the pieces rays or segments? Why?

Segments, because they have endpoints on both ends.

8. Are all the endpoints solid dots or open dots or some of each? Why?

Solid when $\leq$ and open when $<$

9. Were all these x values necessary to graph this piecewise function, or could this have been graphed using less points?

No, we just needed to graph critical points.

10. Which x values were "critical" to include in order to sketch the graph of this piecewise function?

Critical Points: −5, 2, 6,

↳ Twice so we know where to graph both functions at $x=2$.

11. Can you generalize which x-values are essential to input into your table to make a hand sketched graph of a piecewise linear function?

The endpoints of the inequality.

12. Now graph this piecewise function: $f(x) = \begin{cases} x+3 & ,-8 \leq x < 1 \\ 10-2x & ,1 \leq x \leq 7 \end{cases}$

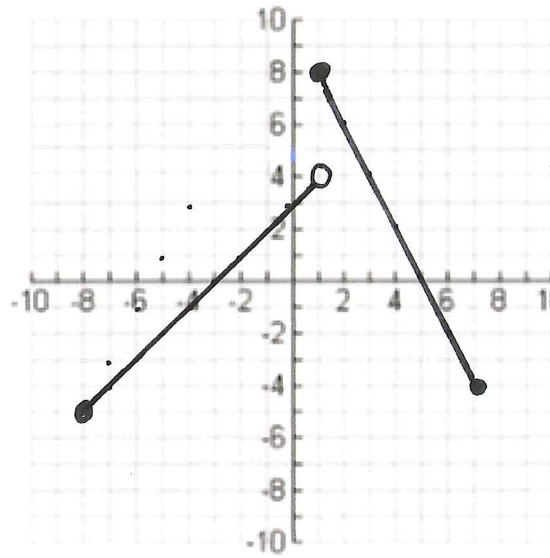
by completing a table of values for the piecewise function over the given domain.

	x	f(x)
●	−8	−5
○	1	4
●	1	8
●	7	−4

13. Why did you choose the x values you placed into the table?

These are the critical points.

14. Graph the ordered pairs from your table to Sketch the graph of the piecewise function.



15. How many pieces does your graph have? Why?

2, because we have 2 equations.

16. Are the pieces rays or segments? Why?

Segments they have endpoint on both ends.

17. Are all the endpoints filled circles or open circles or some of each? Why?

Solid circles when $\leq$ and Open circles when $<$

18. Was it necessary to evaluate both pieces of the function for the x-value 1? Why or why not?

Yes because 1 is a critical point for both equations and we had to graph the open + closed circles at $x=1$.

19. Which x values were "critical" to include in order to graph this piecewise function? Explain.

$-8, 1, 7$ The x-values were the restrictions or end points for each equation.
↳ Twice

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Writing piecewise functions given a graph.

1. Can you identify the equations of the lines that contain each segment?

   a. Left segment equation=

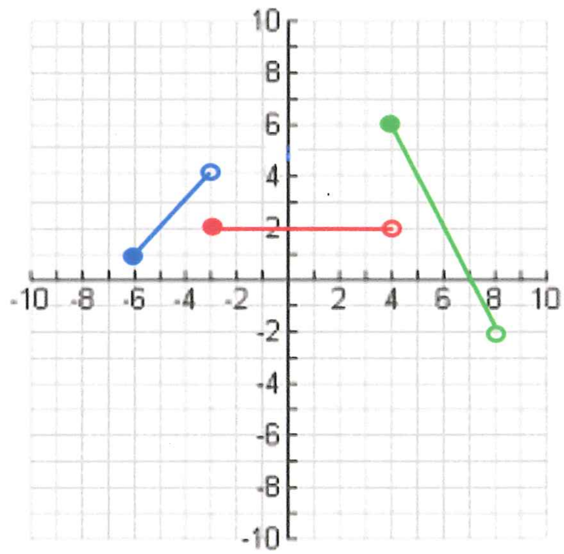
   $x+7$

   b. Middle segment equation=

   2

   c. Right segment equation=

   $-2x+14$



2. Next, list the domain of each segment.

   a. Left segment domain=

   $-6 \le x < -3$

   b. Middle segment domain=

   $-3 \le x < 4$

   c. Right segment domain=

   $4 \le x < 8$

3. Now, put the domain together with the equations to write the piecewise function for the graph.

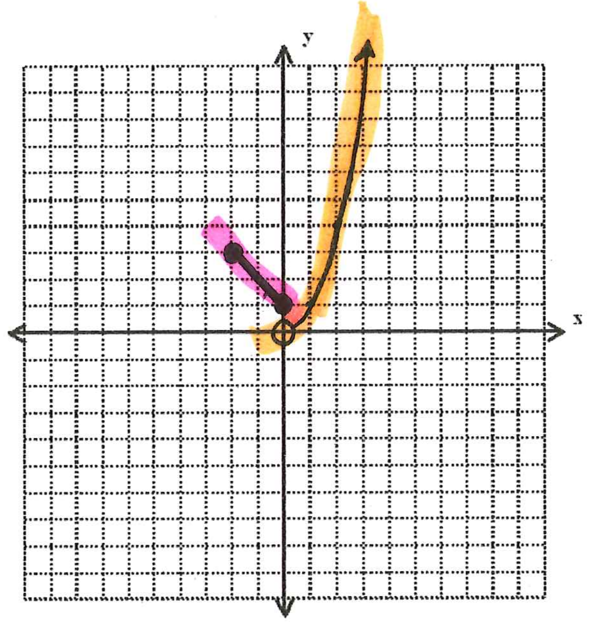
$$f(x) = \begin{cases} x+7, & -6 \le x < -3 \\ 2, & -3 \le x < 4 \\ -2x+14, & 4 \le x < 8 \end{cases}$$

Write a piecewise function for the given graphs.
Example: 1

$$f(x) = \begin{cases} x+1, & -2 \leq x \leq 0 \\ x^2, & x > 0 \end{cases}$$

Segment: Domain: $[-2, 0]$
Equation: $x+1$

Ray: Domain: $(0, \infty)$
Equation: x^2

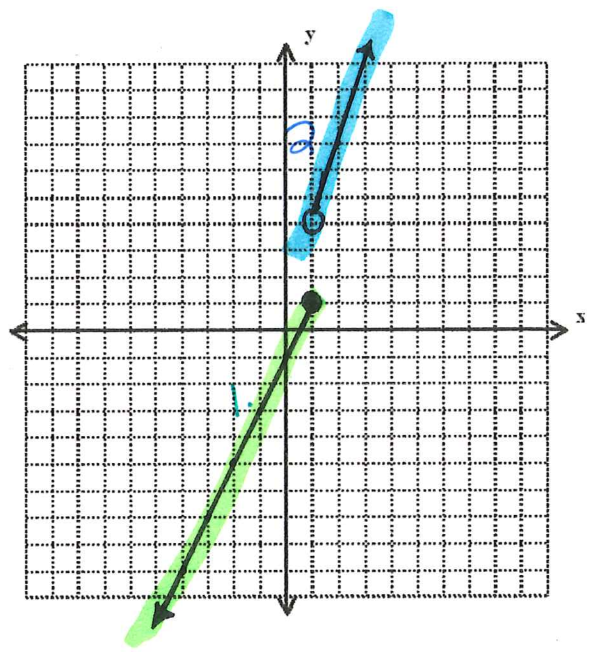


Example 2:

$$f(x) = \begin{cases} 2x-1, & x \leq 1 \\ 3x+1, & x > 1 \end{cases}$$

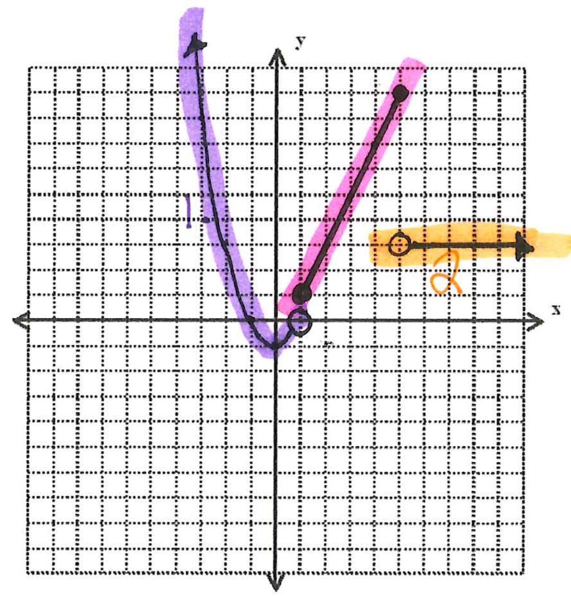
Ray 1: Domain: $[-\infty, 1)$
Equation: $2x-1$

Ray 2: Domain: $(1, \infty)$
Equation: $3x+1$



Example 3:

$$f(x) = \begin{cases} x^2-1, & x<1 \\ 2x-1, & 1 \le x \le 5 \\ 3, & x>5 \end{cases}$$



Ray 1: Domain: $(-\infty, 1)$
Equation: x^2-1

Segment: Domain: $[1,5]$
Equation: $2x-1$

Ray 2: Domain: $(5,\infty)$
Equation: 3

AFM Notes, Unit 4 – Piecewise Functions
4-8 Application of Piecewise Functions

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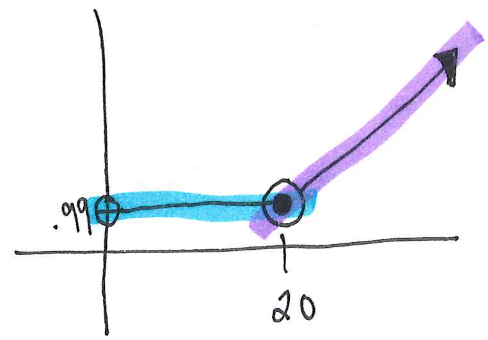
1. A long distance telephone charges 99 cents for any call up to 20 minutes in length and 7 cents for each additional minute. Use bracket notation to write a formula for the cost, C, of a call as a function of its length time, t, in minutes. Graph the function. How much does it cost to talk for 10 minutes? 25 minutes?

C = Cost   t = time

$$c(t) = \begin{cases} 0.99 \;; & [0, 20] \\ 0.99 + 0.07(t-20) ; & t > 20 \end{cases}$$



2. Suppose a carpet store sells carpet for $10 per square yard for the 100 sq yards purchased, and then lowers the price to $7 per square yard after the first 100 yards have been purchased. Find a function, C = f(x), that gives the cost of purchasing any number of square yards of carpet between 0 and 200 square yards. How much does it cost for 50 square yards? 150 square yards?

C = Cost   y = Square yards

$$c(y) = \begin{cases} 10x \,, & [0, 100] \\ 1{,}000 + 7(x-100), & (100, 200] \end{cases}$$

Cost for the 1st 100 yards 10(100) = 1,000

Any amount over 100 yards

$c(50) = 10(50) = 500$

$c(150) = 1{,}000 + 7(x-100)$
$c(150) = 1{,}000 + 7(150-100)$
$c(150) = 1{,}000 + 7(50)$
$c(150) = 1{,}000 + 350$
$c(150) = 1{,}350$

3. A company charges $200 a month to organize a company's payroll for up to 20 employees and an additional $100 a month for each 20 employees over 20. Find a function, P = f(x), that gives the payroll amount for 100 employees in one month. Graph the function.

$$f(x) = \begin{cases} 200, & 0 < x \le 20 \\ 300, & 20 < x \le 40 \\ 400, & 40 < x \le 60 \\ 500, & 60 < x \le 80 \\ 600, & 80 < x \le 100 \end{cases}$$