Math III Unit 2: Polynomials

Notes 2-1 Quadratic Equations

Name: key

Date: ____________ Period: ____________

Solving Quadratic Equations Review

Some quadratic equations can be solved by factoring.
Others can be solved just by using square roots.
ANY quadratic equation can be solved by using quadratic formula.

Solving by Finding Square Roots

a. $4x^2 + 10 = 46$

$$-10 \quad -10$$

$$\frac{4x^2}{4} = \frac{36}{4}$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

b. $3x^2 - 5 = 25$

$$+5 \quad +5$$

$$\frac{3x^2}{3} = \frac{30}{3}$$

$$\sqrt{x^2} = \sqrt{10}$$

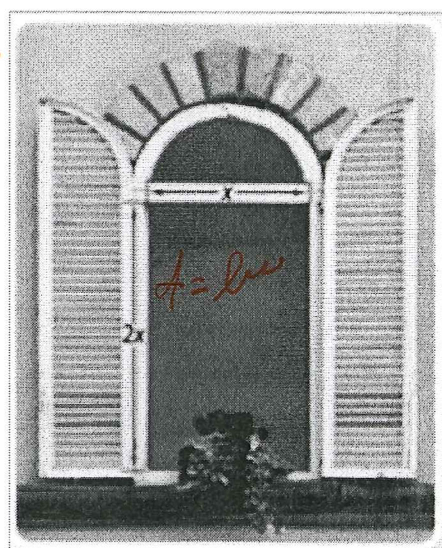
$$x = \pm\sqrt{10}$$

Determining Dimensions

While designing a house, an architect used windows like the one shown here. What are the dimensions of the window if it has 2766 square inches of glass?

$A = lw \qquad \frac{1}{2}A = \frac{1}{2}\pi r^2$

$A = 2x(x) = 2x^2 \qquad A = \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = \frac{1}{2}\pi\frac{x^2}{4} = \frac{\pi}{8}x^2$



$$2x^2 + \frac{\pi}{8}x^2 = 2766$$

$$\frac{x^2\left(2+\frac{\pi}{8}\right)}{\left(2+\frac{\pi}{8}\right)} = \frac{2766}{\left(2+\frac{\pi}{8}\right)}$$

$$\sqrt{x^2} = \sqrt{1156.0167}$$

$$x = \pm 34$$

Solving a Perfect Square Trinomial Equation

What is the solution of $x^2 + 4x + 4 = 25$?

Factor the Perfect Square Trinomial

$x^2 + 4x + 4 = 25$

$(x+2)^2 = 25$

Find Square Roots

$x + 2 = \pm 5$

Rewrite as two equations

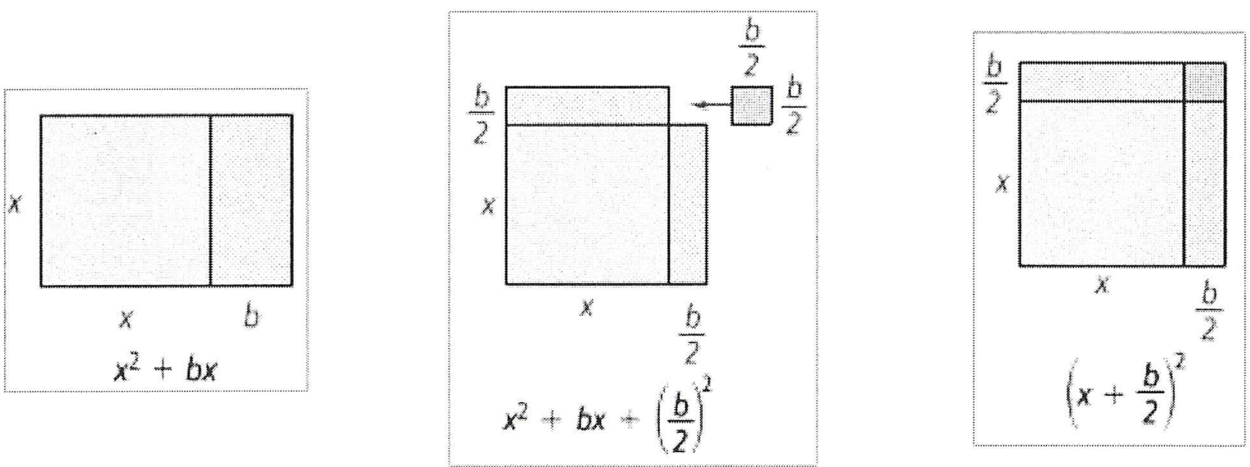
$x+2=5 \qquad x+2=-5$

Solve for x

$x = 3, -7$

Completing the Square

If $x^2 + bx$ is not part of a perfect square trinomial, you can use the coefficient b to find a constant c so that $x^2 + bx + c$ is a perfect square. When you do this, you are ______________ ____ ____________. The diagram models this process.



You can form a perfect square trinomial from $x^2 + bx$ by adding $\left(\frac{b}{2}\right)^2$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example: What value completes the square for $x^2 - 10x$? Justify your answer.

$c = \left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right) = (-5)^2 = 25$

$x^2 - 10x + 25$

$x^2 - 10x + 25 = (x-5)^2$

Solving an Equation by Completing the Square

1. Rewrite the equation in the form $x^2 + bx = c$. To do this, get all terms with the variable on one side of the equation and the constant on the other side. Divide all the terms of the equation by the coefficient of x^2 if it is not 1.
2. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to each side of the equation.
3. Factor the trinomial.
4. Find square roots.
5. Solve for *x*.

Example 1 – What is the solution of $3x^2 - 12x + 6 = 0$?

$3x^2 - 12x + 6 = 0$
$\quad\quad\quad\quad - 6 \quad -6$

$\frac{3x^2 - 12x}{3} = \frac{-6}{3}$

Example 2 – What is the solution of $2x^2 - x + 3 = x + 9$?

$2x^2 - x + 3 = x + 9$

$\frac{2x^2 - 2x}{2} = \frac{6}{2}$

$c = \left(\frac{b}{2}\right)^2 = \left(\frac{-1}{2}\right)^2 = \left(\frac{1}{4}\right)$

$x^2 - x + \frac{1}{4} = 3 + \frac{1}{4}$

$\sqrt{\left(x - \frac{1}{2}\right)^2} = \sqrt{\frac{13}{4}}$

$x = \frac{1}{2} \pm \frac{\sqrt{13}}{2}$

$x - \frac{1}{2} = \pm \frac{\sqrt{13}}{2}$

Writing in Vertex Form

What is $y = x^2 + 4x - 6$ in vertex form? Name the vertex and y-intercept.

$x^2 + 4x + \underline{4} = 6 + 4$ $c = \left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$

$(x+2)^2 = 10$

$y = a(x-h)^2 + k$

$y = 1(x+2)^2 - 10$

Math III Unit 2: Polynomials **Name:** ____________________

Notes 2-2 Real and Imaginary Numbers **Date:** ____________ **Period:** ________

Deriving the Quadratic Formula

$ax^2 + bx + c = 0$

$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0 \} x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide each side by *a*.
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Rewrite so all terms containing *x* are on one side.
	Complete the Square. $c = \left(\frac{b}{2a}\right)^2$
	Factor the perfect square trinomial. Also simplify.
	Find square roots.
	Solve for *x*. Also simplify the radical.
	Simplify.

The Discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac$

- is greater than zero, then two real solutions
- is equal to zero, then one real solution
- is less than zero, then no real solution → two imaginary solutions

Non-real solutions to the quadratic formula are known as imaginary numbers.

Real numbers and imaginary numbers are a subset (a piece) of a larger set of numbers known as complex numbers.

Essential Understanding

The complex numbers are based on a number whose square is -1.

The imaginary unit is the complex number whose square is -1. So, $i^2 = -1$, and $i = \sqrt{-1}$.

Square Root of a Negative Real Number

For any positive number a, $\sqrt{-a} = \sqrt{-1 \bullet a} = \sqrt{-1} \bullet \sqrt{a} = i\sqrt{a}.$

$\sqrt{-5} = i\sqrt{5}$

$\sqrt{-1 \cdot 5}$

$\sqrt{-1}\sqrt{5}$

Note that $\left(\sqrt{-5}\right)^2 = \left(i\sqrt{5}\right)^2 = i^2\left(\sqrt{5}\right)^2 = -1 \bullet 5 = -5$ (not 5).

Simplify a Number Using i

How do you write $\sqrt{-18}$ by using the imaginary unit i?

18

$\boxed{2}$ 9

$\boxed{3\ 3}$

$\sqrt{-18} = \sqrt{-1 \cdot 18}$

$\sqrt{-1} = i$

$= \sqrt{-1} \cdot \sqrt{18}$ Multiplication Property of Square Roots

$= i\sqrt{18}$ Definition of i

$= i\,3\sqrt{2}$ Simplify.

$= 3i\sqrt{2}$

An imaginary number is any number of the form $a + bi$, where a and b are real numbers and $b \neq 0$.

take note

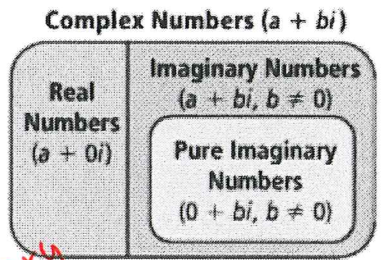
Key Concept Complex Numbers

You can write a **complex number** in the form $a + bi$, where a and b are real numbers.

If $b = 0$, the number $a + bi$ is a real number.

If $a = 0$ and $b \neq 0$, the number $a + bi$ is a **pure imaginary number**.

$\underset{\text{Real part}}{a} + \underset{\text{Imaginary part}}{bi}$



real part

imaginary part

$a + bi$ ← complex #

real #

imaginary unit

$7 - 4i$ → but together they make a complex number

real part

imaginary part

* $8+2i$ complex #

* $6i$ imaginary #

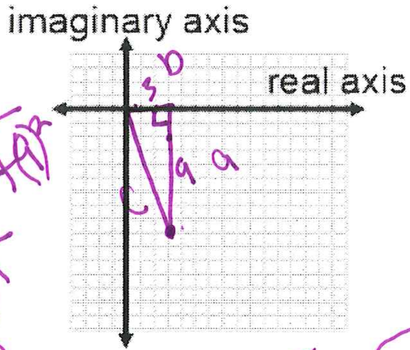
* 9 real #

Complex Number Plane

In the complex number plane, the point (a, b) represents the complex number $a + bi$. To graph a complex number, locate the real part on the horizontal axis and the imaginary part on the vertical axis.

The absolute value of a complex number is its distance from the origin in the complex plane.

$$|a+bi| = \sqrt{a^2+b^2}$$



$|3-9i|$

$= \sqrt{(3)^2+(9)^2}$

$= \sqrt{9+81}$

$= \sqrt{90}$

$= 3\sqrt{10}$

$3-9i$

$\sqrt{c^2} = \sqrt{a^2+b^2}$

$c = \sqrt{a^2+b^2}$

Pythagorean Theorem

$c^2 = a^2 + b^2$

Graphing in the Complex Number Plane

What are the graph and absolute value of each number?

A. $-5+3i$ $(-5, 3)$

$|-5+3i| = \sqrt{a^2+b^2}$

$= \sqrt{(-5)^2+(3)^2} = \sqrt{34}$

$= \sqrt{25+9}$

$= \sqrt{34}$

B. $6i$ $(0, 6)$

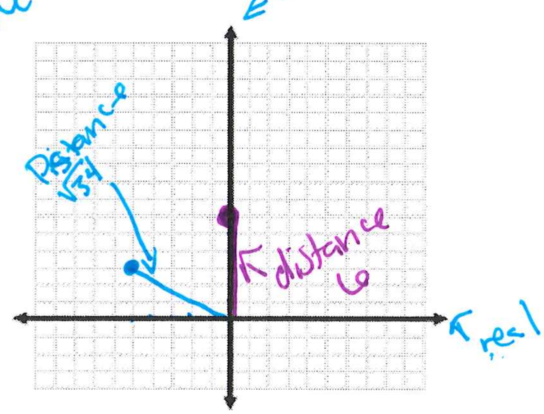
$|6i| = \sqrt{a^2+b^2}$

$= \sqrt{0^2+(6)^2}$

$= \sqrt{36}$

$= 6$

complex plane



Adding and Subtracting Complex Numbers

To add or subtract complex numbers, combine the real parts and the imaginary parts separately. The associative and commutative properties apply to complex numbers.

What is each sum or difference?

A. $(4-3i)+(-4+3i)$

$(4+(-4)) + (-3i+3i)$

$0 + 0$

0

B. $(5-3i)-(-2+4i)$

$(5+(+2)) + (-3i-(4i))$

$7 + (-7i)$

$7-7i$

Multiplying Complex Numbers

You multiply complex numbers $a+bi$ and $c+di$ as you would multiply binomials. For imaginary parts bi and di, $(bi)(di)=bd(i)^2=bd(-1)=-bd$.

Example: What is each product? $i^2=-1$

A. $(3i)(-5+2i)$

$3i(-5)+(3i)(2i)$

$-15i+6i^2$

$-15i+6(-1)$

$-15i-6$

$-6-15i$

B. $(4+3i)(-1-2i)$

$2-11i$

C. $(-6+i)(-6-i)$

$-6(-6-i)+i(-6-i)$

$36+\cancel{6i}\cancel{-6i}-i^2$

$36+(+1)$

37

The solution to this problem is a real number.

Number pairs of the form $a+bi$ and $a-bi$ are complex conjugates.

The product of these types of pairs is a real number.

$$(a+bi)(a-bi)=a^2-(bi)^2=a^2-b^2i^2=a^2-b^2(-1)=a^2+b^2$$

Dividing Complex Numbers

You can use complex conjugates to simplify quotients of complex numbers.

What is each quotient?

A. $\frac{9+12i}{3i}\cdot\frac{3i}{3i}=\frac{3i(9+12i)}{3i(3i)}=\frac{27i-36}{-9}$

$=\frac{27i}{-9}+\frac{-36}{-9}=-3i+4$

$4-3i$

B. $\frac{2+3i}{1-4i}\cdot\frac{1+4i}{1+4i}=\frac{(2+3i)(1+4i)}{(1-4i)(1+4i)}$

$=\frac{2+8i+3i-12}{1+16}=\frac{-10+11i}{17}=\frac{-10+11i}{7}$

Finding Pure Imaginary Solutions

What are the solutions of $2x^2 + 32 = 0$?

$$-32 \quad -32$$

$$\frac{2x^2}{2} = \frac{-32}{2}$$

$$\sqrt{x^2} = \sqrt{-16} \leftarrow \sqrt{-1 \cdot 16}$$

$$\sqrt{-1} \cdot 4$$

$$x = \pm 4i \qquad i \cdot 4$$

$$4i$$

Finding Imaginary Solutions

What are the solutions of $2x^2 - 3x + 5 = 0$?

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 2$ $b = -3$ $c = 5$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 - 40}}{4}$$

$$x = \frac{3 \pm \sqrt{-31}}{4}$$

$$x = \frac{3 \pm i\sqrt{31}}{4}$$

Notes 2-3 Graphing Polynomial Functions

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Each of these functions is a polynomial function:

a) $f(x) = x^3$ b) $f(x) = 7x^2 - 2x^1$ c) $f(x) = 5x^3 + 2x^1 - 12$ d) $f(x) = \frac{3}{4}x^2 - 5x^1 + 7.9$

Each of these functions is NOT a polynomial function: → $x^{\frac{1}{2}}$ ← exponet ← root

e) $f(x) = 23x^{2/3}$ f) $f(x) = 26*(.5)^x$ g) $f(x) = \sqrt{x}$ h) $\frac{1}{x^3}$

A polynomial function has atleast one term
no variables in the exponent
no fractional exponents
no negative exponents

Definition of a Polynomial Function –
A polynomial function is a sum of power functions
whose exponents are positive integers.

SN: power function $y = ax^k$ ← real #s

Standard Form of a Polynomial Function

$g(x) = 3x^2 + 4x^5 + x - x^3 + 6$ is a polynomial function. (Recall that even 6 can be written as $6x^0$.) However, g(x) is not written in standard form. The standard (or general) form for g(x) is $4x^5 - x^3 + 3x^2 + x + 6$.

SN: Anything raised to the zero power is 1.

For a polynomial to be in standard form, the exponents are ordered highest to lowest.

$3x^2 + 4x^5 + x - x^3 + \boxed{6x^0}$

Polynomials have the following characteristics:

a) Degree: The highest power of the exponents.
Ex: Degree 5

b) # of Terms: Number of monomial terms.
Ex: 5 terms

c) Leading Term: The monomial with the highest degree.
Ex: $4x^4$

d) Leading Coefficient: The coefficient of the leading term
Ex: 4

e) Constant Term: The number/term with no variable.
Ex: 6

SN: Coefficient is the number in front of a variable.

Ex 1) Are the following functions polynomial functions? If so, put them in standard form and state a) the degree, b) # of terms, c) leading term, d) leading coefficient, and e) constant term. If not, then tell why not.

a) $y = 3x^2 + 5$
a) 2nd degree
b) 2 terms - Binomial
c) $3x^2$
d) 3
e) 5

b) $y = 4x^2 - 7\sqrt{x^9} + 10$
$y = 4x^2 - 7x^{\frac{9}{2}} + 10$
Not a polynomial b/c there is a fractional exponent.

c) $y = 5^x - 2$
Not a polynomial b/c there is a variable as an expinent.

d) $y = 7t^2 - 8t + 6$
a) 2nd degree
b) 3 terms - trinomial
c) $7t^2$
d) 7
e) 6

e) $y = 3.1 - 8x^2 + 5x^5 - 12.3x^4$
$5x^5 - 12.3x^4 - 8x^2 + 3.1$
a) 5th degree
b) 4 terms
c) $5x^5$
d) 5
e) 3.1

f) $y = x^2 + 5x$
a) 2nd degree
b) 2 - binomial
c) x^2
d) 1
e) ∅

Large-Scale Behavior of Polynomial Functions

Ex.2) Consider the polynomial $f(x) = \boxed{3x^2} + x + 6$.

$3x^2 = 3(2)^2 = 3(4) = 12$

a) What is the value of f(x) when x=2? 20
What is the value of just the leading term when x=2? 12
Notice that when x=2, the value of the leading term makes up 60% of the value of the whole polynomial.
$12/20 = .6 = 60\%$

b) What is the value of f(x) when x=100? 30,106 $3(100)^2 + 100 + 6$
What is the value of just the leading term when x=100? 30,000
Notice that when x=100, the value of the leading term makes up 99.6% of the value of the whole polynomial.
$\frac{30,000}{30,106} = .996$

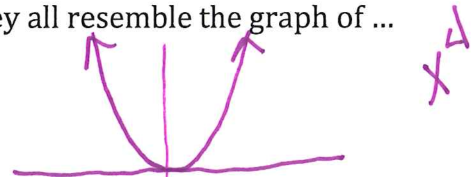
In general, if x is large enough, we can estimate the value of a polynomial by calculating the value of just the leading term.
Thus, for the function f(x), for large values of x, the algebraic expression "$3x^2$" is approximately equal to $3x^2 + x + 6$.

If this is what happens numerically, what would you think would happen graphically?

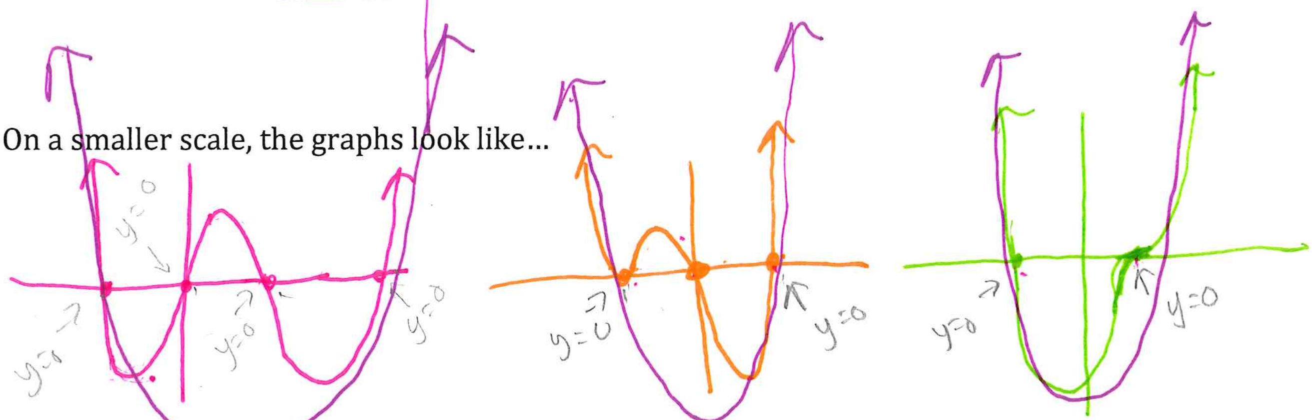
Ex 1: Compare the graphs of the polynomial functions f, g, and h given by…

$f(x) = x^4 - 4x^3 - 4x^2 + 16x$ $g(x) = x^4 + x^3 - 8x^2 - 12x$ $h(x) = x^4 - 4x^3 + 16x - 16$

On a large scale they all resemble the graph of … x^4



On a smaller scale, the graphs look like…



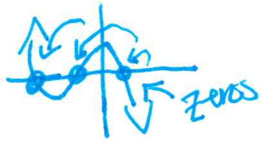
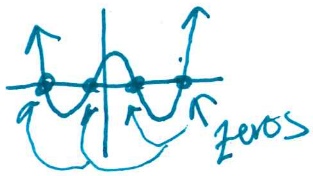
Zeros of a Polynomial Function

The zeros of a function are the values of x that make y equal zero (that is, the values of x that make the function equal zero).
The zeros of a function are also sometimes referred to as:
roots, solutions, or x-intercepts
The **total** number of zeros that a polynomial function has is always equal to the degree of the polynomial. Also note that every zero corresponds to a factor. For example, if x = 2 is a zero then
x-2 is a factor.

Bumps/Turns

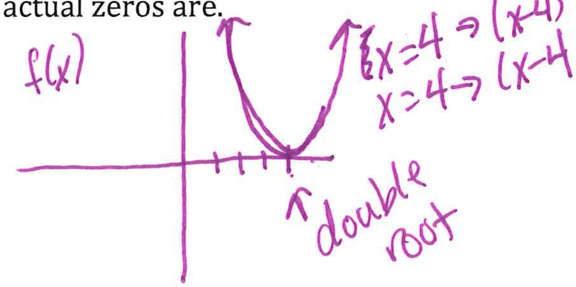
Polynomial functions also have another characteristic that we refer to as bumps or turns.
A bump/turn corresponds to a change in direction for the graph of a function. Between any two consecutive zeros, there must be a bump/turn because the graph would have to change direction or turn in order to cross the x-axis again.

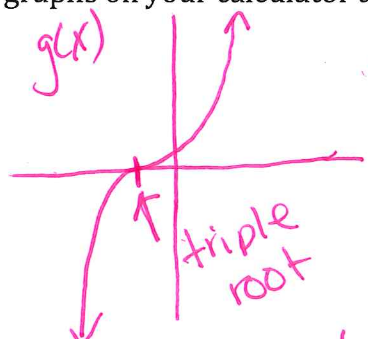
How does the number of bumps/turns compare to the number of zeros?

Type of polynomial	Typical graph shape	# of zeros	# of bumps	
2nd degree quadratic		2	1	16
3rd degree Cubic		3	2	17
4th degree quartic		4	3	18

Multiple Zeros

Consider the functions $f(x) = x^2 - 8x + 16$ and $g(x) = x^3 + 3x^2 + 3x + 1$. How many zeros should f(x) have? 2 What about g(x)? 3 Now look at the graphs on your calculator and determine what the actual zeros are.





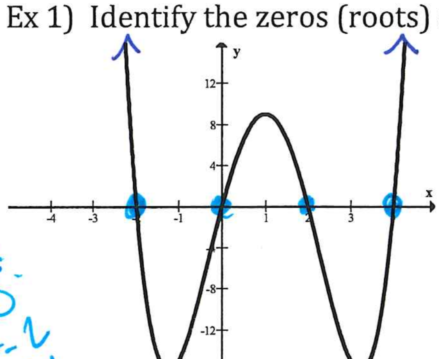
x = -1
x = -1
x = -1
(x+1)(x+1)(x+1)

For f(x) we say that x= 4 is a double root and for g(x) we say that x = -1 is a triple root.

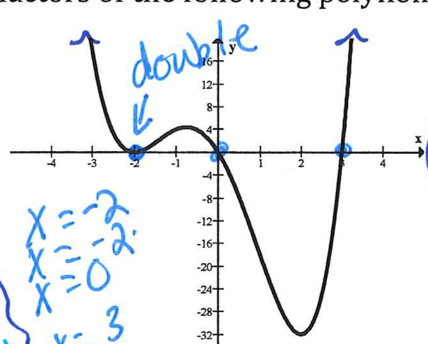
If the graph of a polynomial function bounces off the x-axis, then the zero at that point will be repeated an even number of times.

If the graph of a polynomial function crosses the x-axis but looks flattened there, then the zero at that point will be repeated an odd number of times.

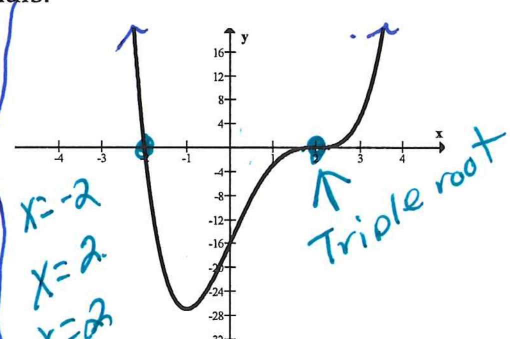
Ex 1) Identify the zeros (roots) and factors of the following polynomials.



x = -2
x = 0
x = 2
x = 4
(x+2)(x)(x-2)(x-4)



x = -2
x = -2
x = 0
x = 3
(x+2)(x+2)(x)(x-3)



x = -2
x = 2
x = 2
x = 2
(x+2)(x-2)(x-2)(x-2)

Polynomials in Factored Form
If we know the zeros of a polynomial function then we can write the polynomial in **factored form,** and can use the factored form to come up with a formula for the polynomial.

Factored Form for a polynomial is f(x) = $(x-r_1)(x-r_2)(x-r_3)\cdots(x-r_n)$ (and so on depending on the number of factors) where a is constant and the "r's" are roots, zeros, solution, or x-intercept.

Example $x = 2, -3, 4, -6$

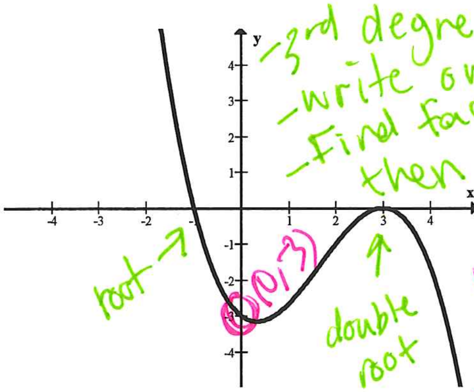
$x=2$, $-2\ -2$, $(x-2)=0$

$x=-3$, $+3\ +3$, $(x+3)=0$

$x=4$, $-4\ -4$, $(x-4)=0$

$x=-6$, $+6\ +6$, $(x+6)=0$

Ex. 2) Find a formula for this polynomial.



- 3rd degree
- write out zeros
- Find factors then simplify

$x=-1$

$x=3$

$x=3$

$(x+1)(x-3)(x-3) \leftarrow$

$y=a(x+1)(x-3)(x-3)$

$-3=a(0+1)(0-3)(0-3)$

$-3=a(1)(-3)(-3)$

$-3=a(9)$

$-3=9a$

$\frac{-3}{9}=\frac{9a}{9}$

$a=-\frac{1}{3}$

$(x^2-3x+x-3)(x-3)$

$(x^2-2x-3)(x-3)$

$x^3\underline{-3x^2-2x^2}+6x-3x+9$

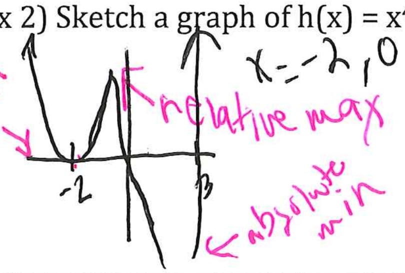
$-\frac{1}{3}(x^3-5x^2+3x+9)$

$-\frac{1}{3}\left(-\frac{1}{3}x^3+\frac{5}{3}x^2-x-3\right)$

Maximum and Minimum Values of a Polynomial Function
Another important characteristic of polynomial functions is their maximum or minimum y-values. Polynomials may have a relative maximum or a relative minimum, which means a maximum or minimum value within a particular range of x-values. They may also have an absolute maximum or an absolute minimum, which means a y-value that is a maximum or minimum over the entire domain of the polynomial.

Ex 2) Sketch a graph of $h(x) = x^4 + x^3 - 8x^2 - 12x$ and identify any maximum or minimum values.

$x = -2, 0, 3$



Math III Unit 2: Polynomials
Notes 2-4 Linear Factors, Zeros, x-intercept

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Finding the Zeros of a polynomial function will help you:

- Factor the polynomial
- Graph the function
- Solve the related polynomial equation

Writing a Polynomial in Factored Form

What is the factored form of $x^3 - 2x^2 - 15x$? * $x = 5$ $x = -3$

$x(x^2 - 2x - 15)$	Factor out the GCF, x.
$x(x-5)(x+3)$	Factor $x^2 - 2x - 15$
Check:	
$x(x-5)(x+3) = x(x^2+3x-5x-15)$	Multiply $(x-5)(x+3)$
$x(x^2-2x-15 = x^3-2x^2-15x$ ✓	Distributive Property

Roots, Zeros, and x-intercepts

The following are equivalent statements about a real number b and a polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$.

- $x-b$ is a linear factor of the polynomial $p(x)$
- b is a zero of the polynomial $p(x)$
- b is the root (solution) of the polynomial $p(x)=0$
- b is an x-intercept of the graph $y=p(x)$.

$\boxed{(x+2)(x-1)}(x-3)$

$(x^2-x+2x-2)(x-3)$

$(x^2+x-2)(x-3)$

$x^3 \boxed{-3x^2} \boxed{+x^2} \boxed{-3x} \boxed{-2x} +6$

x^3-2x^2-5x+6

Finding Zeros of a Polynomial Function

What are the zeros of $y=(x+2)(x-1)(x-3)$?

Use the Zero-Product Property to find the zeros.

$x+2=0$, $-2\ -2$, $x=-2$

$x-1=0$, $+1\ +1$, $x=1$

$x-3=0$, $+3\ +3$, $x=3$

Factor Theorem

The Factor Theorem describes the relationship between the linear factor of a polynomial and the zeros of a polynomial.

Factor Theorem

The expression $x-a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Writing a Polynomial Function From Its Zeros

A. What is a cubic polynomial function in standard form with zeros -2, 2, and 3?

$x=-2$ $x=2$ $x=3$

$(x+2)$ $(x-2)$ $(x-3)$

$(x+2)(x-2)$

$(x^2-4)(x-3)$

$y=x^3-3x^2-4x+12$

B. What is a quartic polynomial function in standard form with zeros $-2,\ -2,\ 2$, and 3?

$x=-2$ $x=-2$ $x=2$ $x=3$

$(x+2)\ (x+2)\ (x-2)\ (x-3)$

$x^2+2x+2x+4$ $x^2-3x-2x+6$

x^2+4x+4 (x^2-5x+6)

$x^2(x^2-5x+6)+4x(x^2-5x+6)+4(x^2-5x+6)$

$x^4-5x^3+6x^2+4x^3-20x^2+24x+4x^2-20x+24$

$x^4-x^3-10x^2+4x+24$

Graph both functions.

1. How do the graphs differ?

Blue (A) – 2 bumps/turns
– Cubic
– relative min + max
– no absolute min or max

Red (B) – 3 bumps/turns
– Quartic
– relative max + min
– absolute min

2. How are they similar?

– Both have a relative max + min.
– They have 3 zeros @ $x=-2, 2, 3$.

Multiple Zeros - When a linear factor, $x-a$, are repeated. The zero, root, or x-intercept occurs multiple times or is repeated.

Zero of multiplicity - Means that $x-a$ appears $\underline{n}$ times as a factor.

Ex x^2+2x+1
$(x+1)(x+1)=(x+1)^2 \leftarrow$ multiplicity $x=-1$ is 2 or a double root

How Multiple Zeros Affect a Graph

If *a* is a zero of multiplicity *n* in the polynomial function $y=p(x)$, then the behavior of the graph at the x-intercept *a* will be:

- Close to linear if $n=1$
- Close to quadratic if $n=2$
- Close to cubic if $n=3$
- Close to quartic if $n=4$

* This will continue on + on

Finding the Multiplicity of a Zero

What are the zeros of $f(x)=x^4-2x^3-8x^2$?

$x^2(x^2-2x-8)$ $\sqrt{x^2}=\sqrt{0}$ $x-4=0$ $x+2=0$
$x^2(x-4)(x+2)$ $x=0,0$ $x=4$ $x=-2$

What are their multiplicities?

0 has a multiplicity of 2 (double root)
4, -2 have multiplicity of one

How does the graph behave at these zeros?

It is close to linear @ $x=-2,4$
It is close to quadratic @ $x=0$

Notes 2-5 Solving Polynomials Equations

Name________________
Date__________ Period _____

To solve a polynomial equation by factoring:

1. Write the equation in the form $P(x)=0$ for some polynomial function P.
2. Factor $P(x)$. Use the Zero Product Property to find the roots.

Solving Polynomial Equations Using Factors

What are the real or imaginary solutions of each polynomial equation?

A. $2x^3 - 5x^2 = 3x$

$2x^3-5x^2-3x=0$	Rewrite in the form P(x) = 0.
$x(2x^2-5x-3)=0$	Factor out the GCF, x.
$x(2x+1)(x-3)=0$	Factor $2x^3 - 5x^2 = 3x$
$x=0$ $2x+1=0$ $x-3=0$	Zero Product Property
$-1\ -1$ $+3\ +3$	Solve each equation for x.
$\frac{2x}{2}=\frac{-1}{2}$ $x=3$	$x=0, -\frac{1}{2}, 3$

$x=-\frac{1}{2}$

B. $3x^4 + 12x^2 = 6x^3$

$3x^4-6x^3+12x^2=0$	Rewrite in the form P(x) = 0.
$x^4-2x^3+4x^2=0$	Multiply by $\frac{1}{3}$ to simplify
$x^2(x^2-2x+4)=0$	Factor out the GCF, x^2
$\sqrt{x^2}=\sqrt{0}$ $x^2-2x+4=0$	Zero Product Property
$x=0,0$ $x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(4)}}{2(1)}$ $x=\frac{2\pm\sqrt{-12}}{2}=\frac{2\pm 2i\sqrt{3}}{2}$ $x=1\pm i\sqrt{3}$	Use the Quadratic Formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
The solutions are 0, $1+i\sqrt{3}$, & $1-i\sqrt{3}$.	

12
1 12
3 4
2 3
$\boxed{2\sqrt{3}}$

SN: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Polynomial Factoring Techniques	
Techniques	**Examples**
Factoring out the GCF Factor out the greatest common factor of all the terms.	$15x^4-20x^3+35x^2$ $=5x^2(3x^2-4x+7)$
Quadratic Trinomials Factor ax^2+bx+c, find the factors with the product $a\cdot c$ & the sum to equal b.	$6x^2+11x-10$ $=(3x-2)(2x+5)$
Perfect Square Trinomials $a^2+2ab+b^2=(a+b)^2$ $(a+b)(a+b)$ $a^2-2ab+b^2=(a-b)^2$ $(a-b)(a-b)$	$x^2+10x+25=(x+5)^2=(x+5)(x+5)$ $x^2-10x+25=(x-5)^2=(x-5)(x-5)$
Difference of Squares $a^2-b^2=(a+b)(a-b)$	$4x^2-4=(2x-2)(2x+2)$
Factor by Grouping $ax+ay+bx+by$ $=a(x+y+b(x+y)$ $=(a+b)(x+y)$	$[x^3+2x^2]-3x-6$ $=x^2(x+2)+(-3)(x+2)$ $=(x^2-3)(x+2)$
Sum of Difference of Cubes $a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$	$8x^3+1=(2x+1)(4x^2-2x+1)$ $8x^3-1=(2x-1)(4x^2+2x+1)$

The sum and difference of cubes is a new factoring technique.

Why it Works

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3+b^3 = a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$	Add 0.
$= a^2(a+b) - ab(a+b) - b^2(a+b)$	Factor out a^2, $-ab$, and b^2.
$(a+b)(a^2 - ab - b^2)$	Factor out $(a+b)$.

Solving Polynomial Equations by factoring

What are the real and imaginary solutions of each polynomial equation?

A. $x^4 - 3x^2 = 4$

$x^4 - 3x^2 - 4 = 0$ $\quad (x^2)^2 - 3(x^2) - 4$	Rewrite in the form P(x) = 0.
$a^2 - 3a - 4 = 0$	Let a = x^2.
$(a-4)(a+1) = 0$	Factor.
$(x^2-4)(x^2+1) = 0$	Replace a with x^2.
$(x+2)(x-2)(x^2+1) = 0$	Factor $x^2 - 4$ as a difference of squares
$x+2=0$ $\quad x-2=0$ $\quad x^2+1=0$ $x=-2$ $\quad x=2$ $\quad -1 \; -1$ $\sqrt{x^2} = \sqrt{-1}$ $x = \pm i$	$x = -2, 2, i, -i$

B. $x^3 = 1$

$a = x$
$b = 1$

$x^3 - 1 = 0$	Rewrite in the form P(x) = 0.
$(x-1)(x^2+x+1) = 0$	Factor the difference of cubes.
$x-1=0$ $+1 \; +1$ $x=1$ $x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2}$ $x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$	$x = 1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$

Your Turn

1. $x^4 = 16$

$x^4 - 16 = 0$
$a^2 - 16 = 0$
$(a+4)(a-4) = 0$
$(x^2+4)(x^2-4) = 0$

$x^2 + 4 = 0$
$-4 \quad -4$
$\sqrt{x^2} = \sqrt{-4}$
$x = \pm 2i$

$x^2 - 4 = 0$
$+4 \quad +4$
$\sqrt{x^2} = \sqrt{4}$
$x = \pm 2$

2. $x^3 = 8x - 2x^2$

$x^3 + 2x^2 - 8x = 0$
$x(x^2 + 2x - 8) = 0$
$x(x+4)(x-2) = 0$

$x = 0$

$x + 4 = 0$
$-4 \quad -4$
$x = -4$

$x - 2 = 0$
$+2 \quad +2$
$x = 2$

3. $x(x^2+8) = 8(x+1)$

$x^3 + 8x = 8x + 8$
$x^3 - 8 = 0$
$(x-2)(x^2+2x+4) = 0$
$x - 2 = 0 \quad x^2 + 2x + 4 = 0$
$x = 2$

$x = \dfrac{-2 \pm \sqrt{4 - 4(1)(4)}}{2}$

$= \dfrac{-2 \pm \sqrt{-12}}{2}$

$= \dfrac{-2 \pm 2i\sqrt{3}}{2}$

$= -1 \pm i\sqrt{3}$

Finding Real Roots by Graphing

What are the real solutions of the equation $x^3 + 5 = 4x^2 + x$?

Use **INTERSECT** feature	Use **ZERO** feature
Set $y_1 = x^3 + 5$ & $y_2 = 4x^2 + x$	Rewrite as $x^3 - 4x^2 - x + 5$
Use INTERSECT	Put in $y_1 =$ and graph
Approximate Points of	Use ZERO Feature to find
Intersection	x-intercepts