 Name: ______________________

Notes 2-1 Quadratic Equations **Date:** ____________ **Period:** ________

Solving Quadratic Equations Review

Some quadratic equations can be solved by ____________.
Others can be solved just by using ________________.
ANY quadratic equation can be solved by using __________________________.

Solving by Finding Square Roots

a. $4x^2 + 10 = 46$

b. $3x^2 - 5 = 25$

Determining Dimensions

While designing a house, an architect used windows like the one shown here. What are the dimensions of the window if it has 2766 square inches of glass?

Solving a Perfect Square Trinomial Equation

What is the solution of $x^2 + 4x + 4 = 25$?

Factor the Perfect Square Trinomial

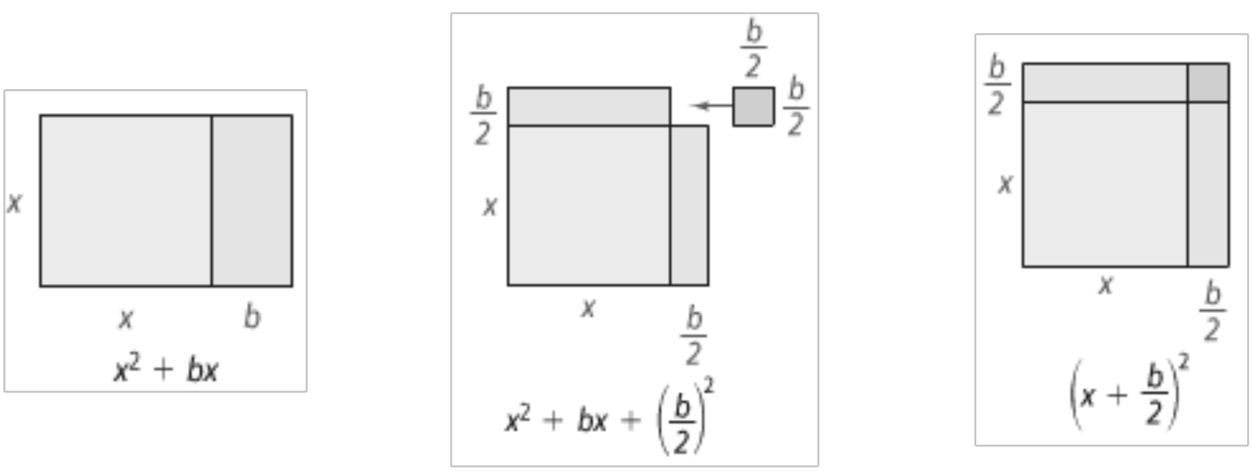
Find Square Roots

Rewrite as two equations

Solve for x

Completing the Square

If $x^2 + bx$ is not part of a perfect square trinomial, you can use the coefficient b to find a constant c so that $x^2 + bx + c$ is a perfect square. When you do this, you are ________________ _____ ______________. The diagram models this process.



You can form a perfect square trinomial from $x^2 + bx$ by adding $\left(\frac{b}{2}\right)^2$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example: What value completes the square for $x^2 - 10x$? Justify your answer.

Solving an Equation by Completing the Square

1. Rewrite the equation in the form $x^2 + bx = c$. To do this, get all terms with the variable on one side of the equation and the constant on the other side. Divide all the terms of the equation by the coefficient of x^2 if it is not 1.
2. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to each side of the equation.
3. Factor the trinomial.
4. Find square roots.
5. Solve for *x*.

Example 1 – What is the solution of $3x^2 - 12x + 6 = 0$?

Example 2 – What is the solution of $2x^2 - x + 3 = x + 9$?

Writing in Vertex Form

What is $y = x^2 + 4x - 6$ in vertex form? Name the vertex and y-intercept.

Math III Unit 2: Polynomials **Name:** ____

Notes 2-2 Real and Imaginary Numbers **Date:** ____________________ **Period:**____________

Deriving the Quadratic Formula

$ax^2 + bx + c = 0$

	Divide each side by *a*.
	Rewrite so all terms containing *x* are on one side.
	Complete the Square.
	Factor the perfect square trinomial. Also simplify.
	Find square roots.
	Solve for *x*. Also simplify the radical.
	Simplify.

The Discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac$

- is greater than zero, then ____________________
- is equal to zero, then ________________________
- is less than zero, then _______________________

Non-real solutions to the quadratic formula are known as ______________ _____________.

Essential Understanding

The complex numbers are based on a number whose square is ____________.

The ___________ _____ is the complex number whose square is -1. So, ________, and ___________.

Square Root of a Negative Real Number

For any positive number a, $\sqrt{-a} = \sqrt{-1 \bullet a} = \sqrt{-1} \bullet \sqrt{a} = i\sqrt{a}.$

$\sqrt{-5} =$ Note that $\left(\sqrt{-5}\right)^2 = \left(i\sqrt{5}\right)^2 = i^2\left(\sqrt{5}\right)^2 = -1 \bullet 5 = -5$ (not 5).

Simplify a Number Using *i*

How do you write $\sqrt{-18}$ by using the imaginary unit i?

Multiplication Property of Square Roots

Definition of i

Simplify.

An _____________ _________ is any number of the form $a + bi$, where a and b are real numbers and $b \neq 0$.

take note

Key Concept Complex Numbers

You can write a **complex number** in the form $a + bi$, where a and b are real numbers.

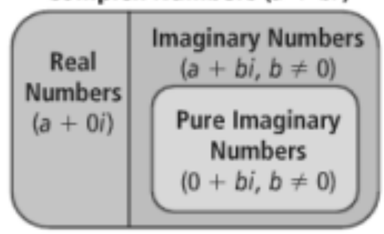
If $b = 0$, the number $a + bi$ is a real number.

If $a = 0$ and $b \neq 0$, the number $a + bi$ is a **pure imaginary number**.

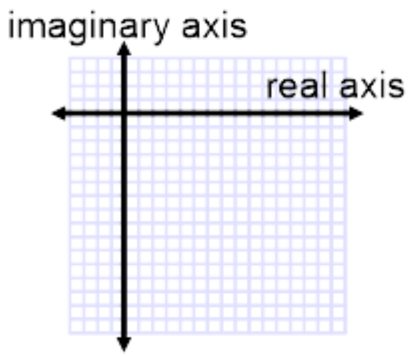
$a + bi$

↑ Real part (a) ↑ Imaginary part (bi)

Complex Numbers ($a + bi$)



Complex Number Plane



In the __________ ________ ______, the point (a, b) represents the complex number $a + bi$. To graph a complex number, locate the ___________ part on the ____________ axis and the ____________ part on the _____________ axis.

The __________ ______ of a complex number is its distance from the origin in the complex plane.

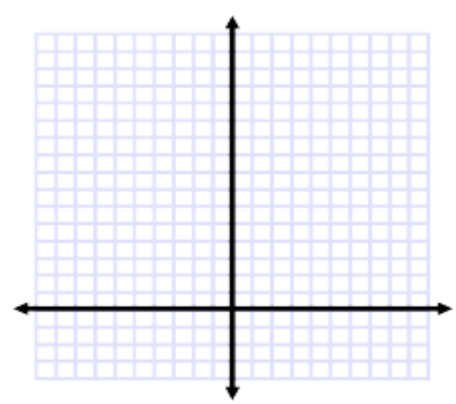
$$|a + bi| = \sqrt{a^2 + b^2}$$

Graphing in the Complex Number Plane

What are the graph and absolute value of each number?

A. $-5 + 3i$

B. $6i$



Adding and Subtracting Complex Numbers

To add or subtract complex numbers, combine the real parts and the imaginary parts separately. The associative and commutative properties apply to complex numbers.

What is each sum or difference?

A. $(4 - 3i) + (-4 + 3i)$

B. $(5 - 3i) - (-2 + 4i)$

Multiplying Complex Numbers

You multiply complex numbers $a+bi$ and $c+di$ as you would multiply binomials. For imaginary parts bi and di, $(bi)(di)=bd(i)^2=bd(-1)=-bd$.

Example: What is each product?

A. $(3i)(-5+2i)$
B. $(4+3i)(-1-2i)$
C. $(-6+i)(-6-i)$ The solution to this problem is a real number.

Number pairs of the form $a+bi$ and $a-bi$ are _______________ _________________.

The product of these types of pairs is a real number.

$$(a+bi)(a-bi)=a^2-(bi)^2=a^2-b^2i^2=a^2-b^2(-1)=a^2+b^2$$

Dividing Complex Numbers

You can use complex conjugates to simplify quotients of complex numbers.

What is each quotient?

A. $\dfrac{9+12i}{3i}$	B. $\dfrac{2+3i}{1-4i}$

Finding Pure Imaginary Solutions

What are the solutions of $2x^2 + 32 = 0$?

Finding Imaginary Solutions

What are the solutions of $2x^2 - 3x + 5 = 0$?

 Name________________________

Notes 2-3 Graphing Polynomial Functions Date______________ Period ______

Each of these functions is a polynomial function:

a) $f(x) = x^3$ b) $f(x) = 7x^2 - 2x$ c) $f(x) = 5x^3 + 2x - 12$ d) $f(x) = \frac{3}{4}x^2 - 5x + 7.9$

Each of these functions is NOT a polynomial function:

e) $f(x) = 23x^{2/3}$ f) $f(x) = 26*(.5)^x$ g) $f(x) = \sqrt{x}$ h) $\frac{1}{x^3}$

A polynomial function ______________________________________

____

____

____

Definition of a Polynomial Function –

A polynomial function is ______________________________________

whose exponents are ______________________________________.

Standard Form of a Polynomial Function

$g(x) = 3x^2 + 4x^5 + x - x^3 + 6$ is a _____________function. (Recall that even 6 can be written as ______.) However, g(x) is not written in standard form. The standard (or general) form for g(x) is ____________________________.

For a polynomial to be in standard form, ________________________________

Polynomials have the following characteristics:

a) Degree:

b) # of Terms:

c) Leading Term:

d) Leading Coefficient:

e) Constant Term:

Ex 1) Are the following functions polynomial functions? If so, put them in standard form and state a) the degree, b) # of terms, c) leading term, d) leading coefficient, and e) constant term. If not, then tell why not.

a) $y = 3x^2 + 5$

b) $y = 4x^2 - 7\sqrt{x^9} + 10$

c) $y = 5^x - 2$

d) $y = 7t^2 - 8t + 6$

e) $y = 3.1 - 8x^2 + 5x^5 - 12.3x^4$

f) $y = x^2 + 5x$

Large-Scale Behavior of Polynomial Functions

Ex.2) Consider the polynomial $f(x) = 3x^2 + x + 6$.

a) What is the value of f(x) when x=2? _______
What is the value of just the leading term when x=2? ______
Notice that when x=2, the value of the leading term makes up _______ of the value of the whole polynomial.

b) What is the value of f(x) when x=100? ______
What is the value of just the leading term when x=100? ______
Notice that when x=100, the value of the leading term makes up _______ of the value of the whole polynomial.

In general, if x is ____________________, we can estimate the value of a polynomial by calculating the value of _______________________________.
Thus, for the function f(x), for large values of x, the algebraic expression "$3x^2$" is approximately equal to $3x^2 + x + 6$.

If this is what happens numerically, what would you think would happen graphically?

Ex 1: Compare the graphs of the polynomial functions f, g, and h given by…

$f(x) = x^4 - 4x^3 - 4x^2 + 16x$ $g(x) = x^4 + x^3 - 8x^2 - 12x$ $h(x) = x^4 - 4x^3 + 16x - 16$

On a large scale they all resemble the graph of …

On a smaller scale, the graphs look like…

Zeros of a Polynomial Function

The zeros of a function are the values of ____ that make ____ equal zero (that is,
the values of _____ that make the __________ equal zero).
The zeros of a function are also sometimes referred to as:

The **total** number of zeros that a polynomial function has is always equal to the _______________of the polynomial. Also note that every _________ corresponds to a _______________. For example, if x = 2 is a ____________ then
x-2 is a _______________.

Bumps/Turns

Polynomial functions also have another characteristic that we refer to as bumps or turns.
A _______________ corresponds to a change in direction for the graph of a function. Between any two consecutive ____________, there must be a ____________because the graph would have to change direction or turn in order to cross the x-axis again.

How does the number of bumps/turns compare to the number of zeros?

Type of polynomial	Typical graph shape	# of zeros	# of bumps
2^{nd} degree			
3^{rd} degree			
4^{th} degree			

Multiple Zeros

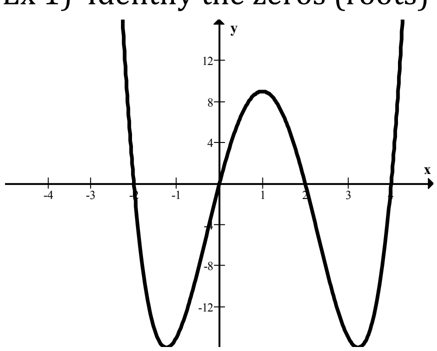
Consider the functions $f(x) = x^2 - 8x + 16$ and $g(x) = x^3 + 3x^2 + 3x + 1$. How many zeros should f(x) have? _____ What about g(x)? _____ Now look at the graphs on your calculator and determine what the actual zeros are.

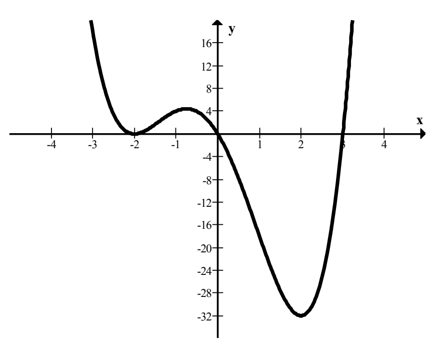
For f(x) we say that x=_____is a ____________ root and for g(x) we say that x =___ is a ____________ root.

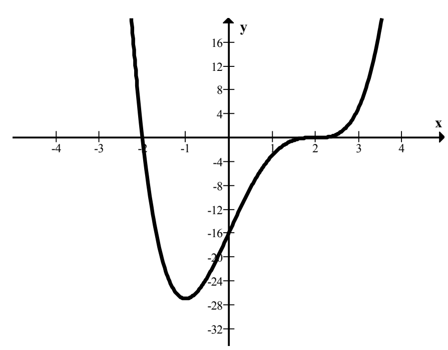
If the graph of a polynomial function ___________ off the x-axis, then the zero at that point will be repeated an _______________ number of times.

If the graph of a polynomial function crosses the x-axis but looks _______________ there, then the zero at that point will be repeated an ______________number of times.

Ex 1) Identify the zeros (roots) and factors of the following polynomials.



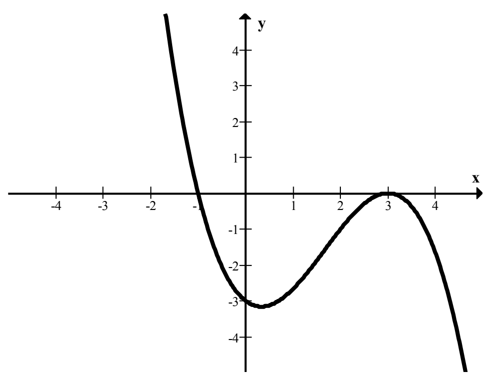




Polynomials in Factored Form
If we know the zeros of a polynomial function then we can write the polynomial in **factored form,** and can use the factored form to come up with a formula for the polynomial.

Factored Form for a polynomial is f(x) = ___________________________________
(and so on depending on the number of factors) where a is _________________
and the “r’s” are ___________

Ex. 2) Find a formula for this polynomial.



Maximum and Minimum Values of a Polynomial Function
Another important characteristic of polynomial functions is their maximum or minimum y-values. Polynomials may have a __________ maximum or a _____________ minimum, which means a maximum or minimum value within a particular range of x-values. They may also have an ________________ maximum or an ________________ minimum, which means a y-value that is a maximum or minimum over the entire domain of the polynomial.

Ex 2) Sketch a graph of $h(x) = x^4 + x^3 - 8x^2 - 12x$ and identify any maximum or minimum values.

Notes 2-4 Linear Factors, Zeros, x-intercept

Name________________________

Date_____________ Period ______

Finding the ________ of a polynomial function will help you:

-
-
-

Writing a Polynomial in Factored Form

What is the factored form of $x^3 - 2x^2 - 15x$?

	Factor out the GCF, x.
	Factor $x^2 - 2x - 15$
Check:	
	Multiply $(x-5)(x+3)$
	Distributive Property

Roots, Zeros, and x-intercepts

The following are equivalent statements about a real number *b* and a polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$.

-
-
-
-

Finding Zeros of a Polynomial Function

What are the zeros of $y=(x+2)(x-1)(x-3)$?

Use the Zero-Product Property to find the zeros.

Factor Theorem

The Factor Theorem describes the relationship between the _________ __________ of a polynomial and the ________ of a polynomial.

<u>**Factor Theorem**</u>

The expression $x-a$ is a factor of a polynomial if and only if the value *a* is a zero of the related polynomial function.

Writing a Polynomial Function From Its Zeros

A. What is a cubic polynomial function in standard form with zeros -2, 2, and 3?

B. What is a quartic polynomial function in standard form with zeros $-2,\ -2,\ 2, \text{ and } 3$?

Graph both functions.

1. How do the graphs differ?

2. How are they similar?

Multiple Zeros –

Zero of multiplicity –

How Multiple Zeros Affect a Graph

If *a* is a zero of multiplicity *n* in the polynomial function _________, then the ___________ of the graph at the x-intercept *a* will be:

-
-
-
-

Finding the Multiplicity of a Zero

What are the zeros of $f(x) = x^4 - 2x^3 - 8x^2$?

What are their multiplicities?

How does the graph behave at these zeros?

Name________________________

Notes 2-5 Solving Polynomials Equations

Date______________ Period ______

To solve a polynomial equation by factoring:

1. Write the equation in the form _________________ for some polynomial function P.
2. Factor _________. Use the Zero Product Property to find the __________.

Solving Polynomial Equations Using Factors

What are the real or imaginary solutions of each polynomial equation?

A. $2x^3 - 5x^2 = 3x$

	Rewrite in the form P(x) = 0.
	Factor out the GCF, x.
	Factor $2x^3 - 5x^2 = 3x$
	Zero Product Property
	Solve each equation for x.

B. $3x^4 + 12x^2 = 6x^3$

	Rewrite in the form P(x) = 0.
	Multiply by $\frac{1}{3}$ to simplify
	Factor out the GCF, x^2
	Zero Product Property
	Use the Quadratic Formula

Polynomial Factoring Techniques	
Techniques	**Examples**
Factoring out the GCF	
Quadratic Trinomials	
Perfect Square Trinomials	
Difference of Squares	
Factor by Grouping	
Sum of Difference of Cubes	

The sum and difference of cubes is a new factoring technique.

Why it Works

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

	Add 0.
	Factor out a^2, $-ab$, and b^2.
	Factor out $(a+b)$.

Solving Polynomial Equations by factoring

What are the real and imaginary solutions of each polynomial equation?

A. $x^4 - 3x^2 = 4$

	Rewrite in the form P(x) = 0.
	Let a = x^2.
	Factor.
	Replace a with x^2.
	Factor $x^2 - 4$ as a difference of squares

B. $x^3 = 1$

	Rewrite in the form P(x) = 0.
	Factor the difference of cubes.

Your Turn

1. $x^4 = 16$

2. $x^3 = 8x - 2x^2$

3. $x\left(x^2 + 8\right) = 8\left(x + 1\right)$

Finding Real Roots by Graphing

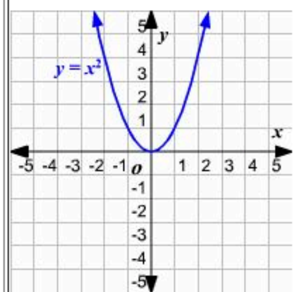
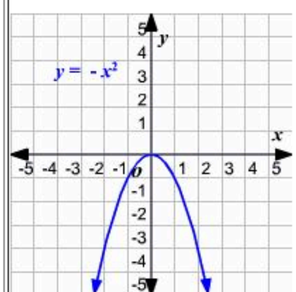
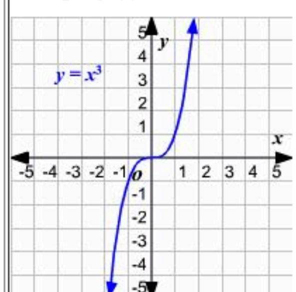
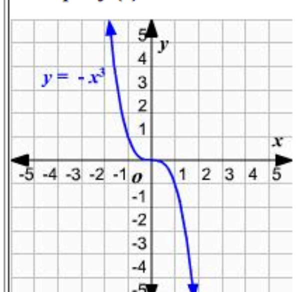
What are the real solutions of the equation $x^3 + 5 = 4x^2 + x$?

Use **INTERSECT** feature	Use **ZERO** feature

Notes 2-6 Power Functions

Name____________________________

Date________________ Period ______

Degree	Leading Coefficient	End behavior of the function	Graph of the function
Even	Positive	$f(x) \to +\infty$, as $x \to -\infty$ $f(x) \to +\infty$, as $x \to +\infty$	Example: $f(x) = x^2$ 
Even	Negative	$f(x) \to -\infty$, as $x \to -\infty$ $f(x) \to -\infty$, as $x \to +\infty$	Example: $f(x) = -x^2$ 
Odd	Positive	$f(x) \to -\infty$, as $x \to -\infty$ $f(x) \to +\infty$, as $x \to +\infty$	Example: $f(x) = x^3$ 
Odd	Negative	$f(x) \to +\infty$, as $x \to -\infty$ $f(x) \to -\infty$, as $x \to +\infty$	Example: $f(x) = -x^3$ 

The degree of a Power Function is _________________________________
____
_________________________________.

The leading coefficient is ________
____
_________________________________.

The end behavior of a power function _________________________
____
____
____
____
_________________________________.

Constant: ______________________

Linear: ________________________

Quadratic: _____________________

Cubic: ________________________

Quartic: _______________________

Quintic: _______________________

Radical: _______________________

Power Functions

Each of these functions is a power function:

a) $f(x) = x^{-3}$ b) $f(x) = -7x^2$ c) $f(x)=23x^{\frac{2}{3}}$ d) $f(x)=\frac{3}{4}x$ e) $f(x)=\sqrt{x}$

Each of these functions is NOT a power function:

f) $f(x) = 5x^3 + 2x$ g) $f(x) = 26(.5)^x$

A power function ____

A power function ____

Definition of a Power Function – a power function has the form ____________,
where k & p are ______________
and k & p can be ______________, ________________, or ___________________.

Ex 1: Which of the following are power functions? If it is a power function, state the value of k and p. If it is not a power function, explain why.

a) $f(x)=13\sqrt[3]{x}$

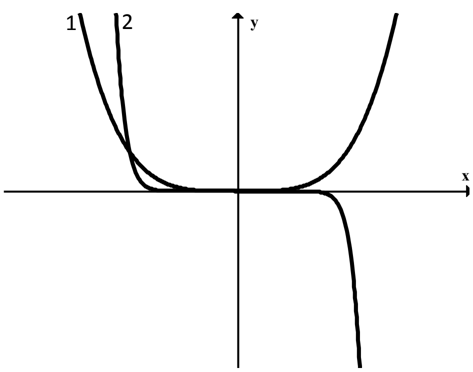
b) $g(x) = 2(x+5)^2$

c) $h(x) = (x+3)(x-3) + 9$

d) $u(x)=\sqrt{\frac{25}{x^3}}$

e) $v(x) = 6 \cdot 3^x$

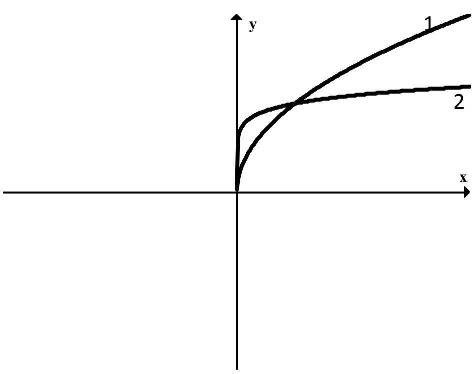
Ex 2. Below are the graphs of two power functions.



a) Which graph would match the power function $f(x) = 3x^4$ and which would match the power function $g(x) = -2x^7$?

b) Name two other power functions which would also match the shape of these two graphs.

Ex. 3) Below are the graphs of two power functions.



a) Which graph would match the power function $f(x) = x^{\frac{1}{2}}$ and which would match the power function $g(x) = x^{\frac{1}{8}}$?

b) Name two other power functions which would also match the shape of these two graphs.

Name_________________________
Date_____________ Period ______

Dividing Polynomials Using Long Division

We will be doing a quick review of long division since we need to know this when working with rational functions. To divide using long division we do the same steps as if we are working with numbers.

EXAMPLE: Divide by using long division: 3453 / 13.

EXAMPLE: Divide by using long division: $\left(2x^2+3x-35\right)\div(x+5)$

EXAMPLE: Divide by using long division: $\left(3x^3+x+5\right)\div(x+1)$

EXAMPLE: Divide by using long division: $\left(x+3x^3-x^2-2\right)\div(x^2+2)$

EXAMPLE: Divide by using long division: $\left(-3x^4 - 2x - 1\right) \div (x-1)$

Synthetic Division - The Shortcut for Dividing by (x – c)
When dividing a polynomial f(x) by a linear factor (x-c), we can use a shorthand notation. saving steps and space. Here is the procedure:

Procedure For Synthetic Division of f(x) by (x – c):

1. Write the value of "c" and the coefficients of f(x) in a row. For example, if we divided $f(x) = 3x^3 + 2x - 1$ by (x – 4) we would write

4	3	0	2	-1

2. Carry down the first coefficient. In this case carry down the 3.

4	3	0	2	-1
	3			

3. Multiply this carried down coefficient by the value of c.
   In this case, multiply 3 • 4 = 12. Place this result in the next column.

4	3	0	2	-1
		12		
	3			

4. Add the column entries and place result at bottom. In this case you add 0+12 to get 12. Multiply this addition result by "c" and place in next column. In this case you multiply 12 • 4 = 48.

4	3	0	2	-1
		12	48	
	3	12		

5. Repeat Step 4 for all columns. In this example, you get

4	3	0	2	-1
		12	48	200
	3	12	50	199

6. The bottom row of numbers reveals the answer along with the remainder. In this case, the numbers **3 12 50 199** indicate an answer of
   $3x^2 + 12x + 50 \quad r\,199$ or $3x^2 + 12x + 50 + 199/(x-4)$

Math III Unit 2: Polynomials

Notes 2-8 Polynomial Theorems

Name________________________

Date______________ Period ______

There are ______ theorems to know about _______________________:

1. _____________________ ______________ _____________________
2. _____________________ ______________ _____________________
3. _____________________ ______________ _____________________
4. _____________________ ______________

Consider the following:

Example 1: $x^3 - 5x^2 - 2x + 24 = 0$

This equation factors to:

The roots therefore are: ______, _______, _______

What do you notice?

Example 2: $24x^3 - 22x^2 - 5x + 6 = 0$

This equation factors to:

The roots therefore are: ______, _______, _______

What do you notice?

Both the ________________ ____________ and the _______________ ________________ of a polynomial can play a key role in identifying the ___________________ ______________ or the related polynomial equation.

This role is expressed in the ________________ ___________ ______________

If ______ is in simplest form and is a rational root of the polynomial equation

$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0 x^0 = 0$, with ____________ _________________,

then ____ must be a factor of _____ and ______ must be a factor of _______.

So for any polynomial:

If ____,

then ____,

and ____.

Example 3: Finding Rational Roots

Find the rational roots of $x^3 + x^2 - 3x - 3 = 0$

Step 1: List the possible rational roots.

The leading coefficient is _____. The constant term is _________. By the Rational Root Theorem, the only possible roots of the equation have the form

__________________.

The factors of ______ are ________ and ________. The factors of _____ are ______. The only possible rational roots are ___________ and ________.

Step 2: Test each possible rational root.

Test ___:

Test ___:

Test ____:

Test____:

YOU TRY: Find the rational root s of $x^3 - 4x^2 - 2x + 8 = 0$

Example 4: Using the Rational Root Theorem

Find the roots of $2x^3 - x^2 + 2x - 1 = 0$.

Step 1: List the possible rational roots.

Step 2: Test each possible rational root until you find the root.

Test ___:

Test ___:

Step 3: Use synthetic division with the root you found in Step 2 to find the quotient.

Step 4: Find the roots of $2x^2 + 2 = 0$

Conjugates: ____

Irrational Root Theorem:

Let ____ and ____ be rational numbers and let _______ be an ______________ number. If _________________ is a root of a polynomial equation with _________________ ______________ then the conjugate _____________ is also a root.

Example 5: Finding Irrational Roots

A polynomial equation with integer coefficients has the roots ________________ and ____________.

Find two additional roots.

By the _________________ _________ _____________, if ______________ is a root then, its conjugate _________________ is also a root. If __________ is a root, then its conjugate _______ also is a root.

YOU TRY: Find the Irrational Roots

A polynomial equation with rational coefficients has the roots $2 - \sqrt{7}$ and $\sqrt{5}$. Find two additional roots.

Imaginary Root Theorem – If the _____________ number _________ is a root of a polynomial with ____________ ________________, then the conjugate ____________ also is a root.

What theorem have we already studied that sounds like the Imaginary Root Theorem? ______________________________

If a _________________ P can be written as a product of its linear factors, $P(x) = a(x - r_1)(x - r_2)\ldots(x - r_n) = 0$ then $r_1, r_2, \ldots, r_n$ are roots of P(x) = 0.

If we were to FOIL out a polynomial with real coefficients with imaginary roots, it would be a polynomial with not imaginary numbers.

$$(x - [a + bi])(x - [a - bi])$$

Example: If a polynomial with real coefficients has $5, 7i, and \sqrt{5} - i$ among its roots, name at least two other roots.

Descartes' Rule of Signs

Let P(x) be a polynomial with real _____________ written in _______________ form.

The number of ___________ real roots of P(x)=0 is either equal to the number of sign changes between consecutive coefficients of P(x) or less than that by an even number.

The number of ___________ real roots of P(x) = 0 is either equal to the number of sign changes between consecutive coefficients of P(-x) or less than that by an even number.

A possible number of positive real roots for

$$x^4 - x^3 + x^2 - x + 1$$

Example $2x^4 - x^3 + 3x^2 - 1 = 0$

Possible Positive Real Roots:

Possible Negative Real Roots:

Math III Unit 2: Polynomials Name________________________
Notes 2-9 Function Notation/Function Operations Date______________ Period ______

Function Operations

You can ________, ____________, ____________, and ____________ functions based on how you perform these operations for real numbers. One difference is that you must consider the _________ of each function.

Function Operations	
Addition	
Subtraction	
Multiplication	
Division	
The domains of the sum, difference, product, and quotient functions consist of the *x*-values that are in the domains of *both f* and *g*. The domain of the quotient does not contain any *x*-value for which *g(x)* = 0.	

Adding and Subtracting Functions

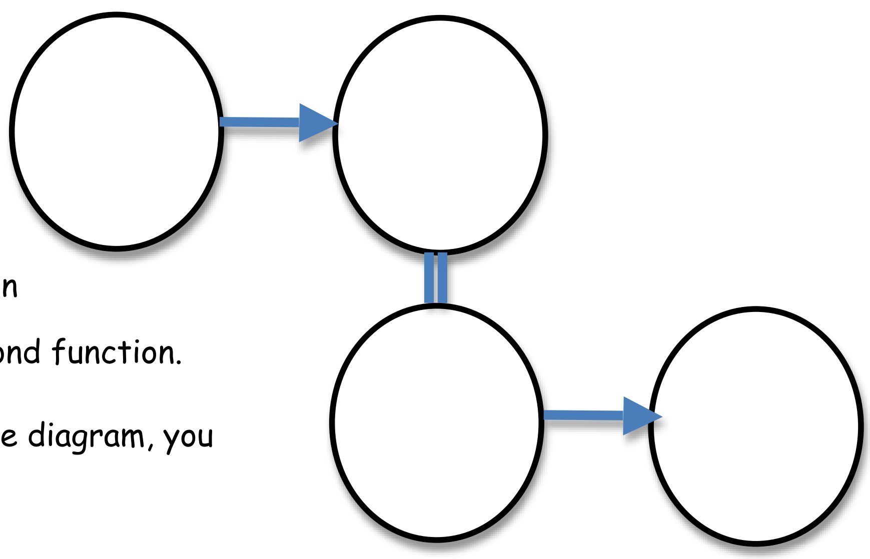
Let $f(x) = 4x + 7$ and $g(x) = \sqrt{x} + x$. What are $f + g$ and $f - g$? What are their domains?

Multiplying and Dividing Functions

Let $f(x)=x^2-9$ and $g(x)=x+3$. What are $f \bullet g$ and $\frac{f}{g}$ and their domains?

Composite Function

The diagram shows what happens when you apply one function *g(x)* after another function *f(x)*.



The ___________ from the first function becomes the ____________ for the second function.

When you combine two functions as in the diagram, you form a _____________ ___________.

Composition of Functions

The composition of function *g* with function *f* is written as $f \circ g$ and is defined as $(g \circ f)(x) = g(f(x))$.

The domain of $g \circ f$ consists of the *x*-values in the domain of *f* for which *f(x)* is in the domain of *g*.

$$g \circ f = g(f(x))$$

Composing Functions

Let $f(x)=x-5$ and $g(x)=x^2$. What is $(g \circ f)(-3)$?

Method 1	Method 2

Additional Examples:

$$f(x)=2x+5$$
$$g(x)=x^2-3x+2$$

1. $-2g(x)+f(x)$

2. $4f(x)+2g(x)$

3. $\dfrac{5f(x)}{g(x)}$

$$f(x) = 2x$$
$$g(x) = x^2 = 4$$

Find the following:

1. $(g \circ f)(1)$

2. $(g \circ f)(-5)$

3. $(g \circ g)(a)$

4. $(f \circ f)(a)$

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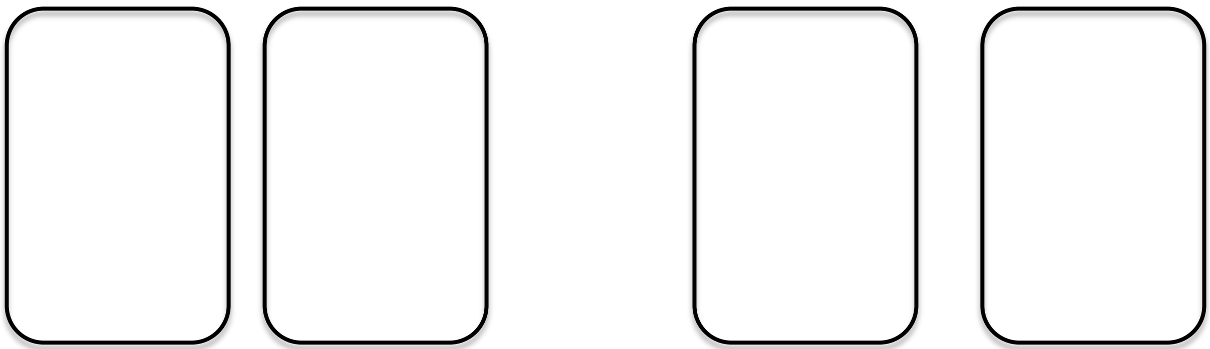
Notes 2-10 Inverse Functions Name________________________ Date______________ Period ______

If a _______________ pairs elements ___ of its domain to element ____ of its range, the _________________ _____________ pairs b with a. So, (*a*,*b*) is an ordered pair of a relation, then (*b*,*a*) is an ordered pair of an inverse.

The diagram shoes a relation r and its inverse.

Relation r Inverse of r



The range of the relation is the domain of the inverse, and the domain of the relation is the range of the inverse.

Example 1: Finding the Inverse of a Relation

a. Find the inverse of relation s.

Relation s

x				
y				

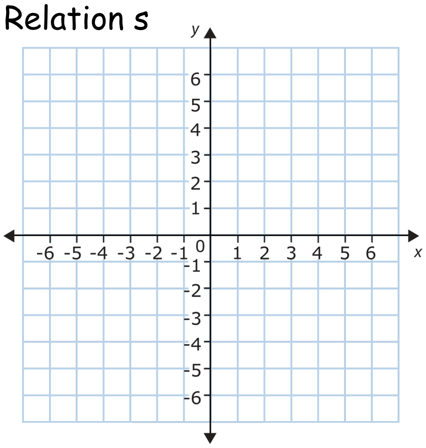
Inverse of Relation s

x				
y				

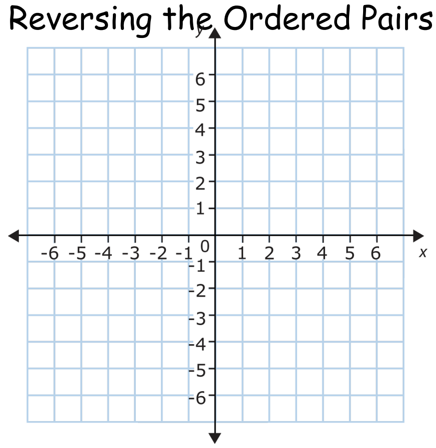
________________ the x and y values to get the _______________.

b. Graph s and its inverse.

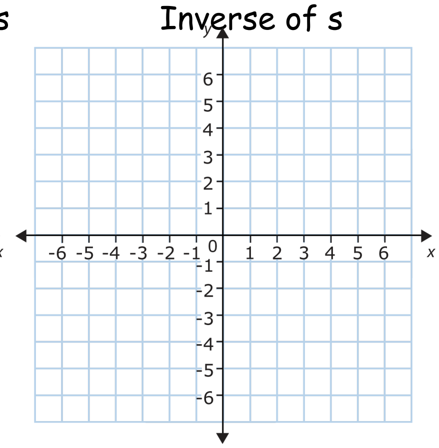
Relation s



Reversing the Ordered Pairs



Inverse of s

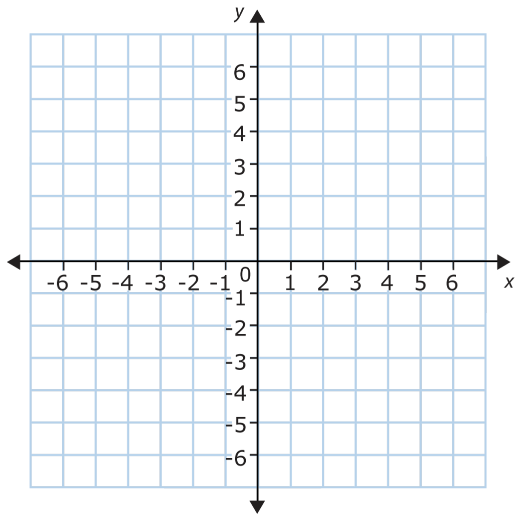


YOU TRY:

Find the Inverse!

Graph the relation and it's inverse.

x	-1	0	1	2
y				



Example 2: Interchanging x and y

Find the inverse of $y = x^2 + 3$

a. Does $y = x^2 + 3$ define a function? Is its inverse a function? Explain.

b. Find the inverse of $y = 3x - 10$. Is the inverse a function? Explain.

Example 3: Finding an Inverse Function

Let's take a look at the function, $f(x) = \sqrt{x+1}$

a. Find the domain and the range of *f(x)*.

b. Find f^{-1}.

c. Find the domain and range of f^{-1}.

d. If f^{-1} a function? Explain.

Example 4: Real-World Connection

The function $d = \frac{r^2}{24}$ is a model for the distance d in feet that car with locked brake skids in coming to a complete stop from a sped of r mi/h. Find the inverse pf the function. What is the best estimate of the speed of a car that made a skid mark 114 feet long?