**Name:**____________________

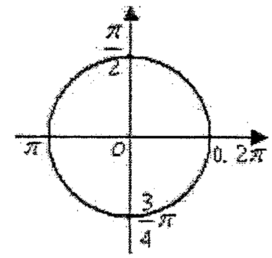
# Angle and Radian Measure

**Measurements of Angles:** Until now you have measured angles in degrees. Another unit for measuring angles is ______________.

**Radians:**

-The circumference of a circle with radius 1 is ___________ so a complete revolution has made __________ radians.

-A straight angle (or _________ of a circle) has measure ____________ radians.




## Converting Radians and Degrees:

$$\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times \text{degrees}$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times \text{radians}$$

## Examples:

1. Express 60° in radians

2. Express $\frac{\pi}{6}$ rad in degrees

**Practice:**

**#1-8, change the given angle to radians.**

1) 315°

2) -60°

3) 212°

4) -168°

5) 12.5°

6) -310°

7) 600°

8) -720°

**#9-16, change the given angle to degrees.**

9) $\frac{3\pi}{4}$

10) $-\frac{9\pi}{5}$

11) $\frac{15\pi}{8}$

12) $-\frac{\pi}{10}$

13) $\frac{7\pi}{10}$

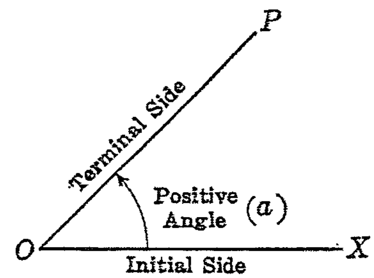
14) $-\frac{16\pi}{15}$

15) $\frac{88\pi}{9}$

16) $-\frac{29\pi}{12}$

# Angles in Standard Position

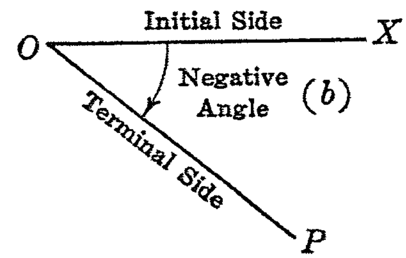
Angle:




Initial Side:

Terminal Side:

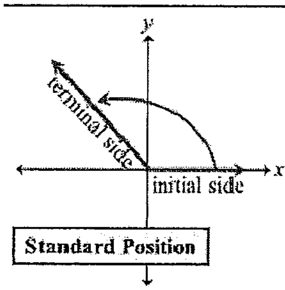
Positive Angle:




Negative Angle:

An angle is in ____________________ if it is draw in the xy-plane with its ____________ at the

________ and its initial side on the ______________________.



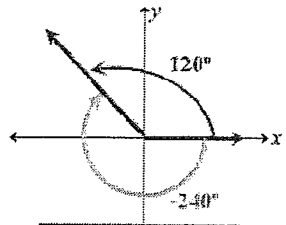

Example: Draw the given angle in standard position. State the quadrant the terminal side is in.

1. $45^0$ 2. $225^0$ 3. $270^0$ 4. $-60^0$

5. $750^0$ 6. $-150^0$ 7. $180^0$ 8. $-75^0$

Coterminal Angles

Two angles in ________________ are ________________ if their sides coincide.



Coterminal Angles

To find angles that are coterminal, add any multiple of _________ for degrees or ____________ for radians.

Examples:

1. Find three angles that are coterminal with the angle $\theta = 30°$ in standard position
2. Find three angles that are coterminal with the angle $\theta = \frac{\pi}{3}$ in standard position
3. Find an angle with a measure between 0° and 360° that is coterminal with the angle of measure 1290° in standard position.

**Find a coterminal angle between 0° and 360°.**

13) $-330°$

14) $-435°$

15) $640°$

16) $-442°$

**Find a coterminal angle between 0 and 2π for each given angle.**

17) $\frac{11\pi}{3}$

18) $-\frac{35\pi}{18}$

19) $\frac{15\pi}{4}$

20) $-\frac{19\pi}{12}$

**Find a positive and a negative coterminal angle for each given angle.**

21) $\frac{5\pi}{4}$

22) $\frac{25\pi}{36}$

3-8 Notes

## Graphs of Sine and Cosine

From your discovery activity yesterday, you should have discovered that sine and cosine values repeat themselves. Thus, the sine and cosine functions are ________________.

A function is ________________ if there is a positive number p such that f(t + p) = f(t) for every t. The least such positive number is called the ________.

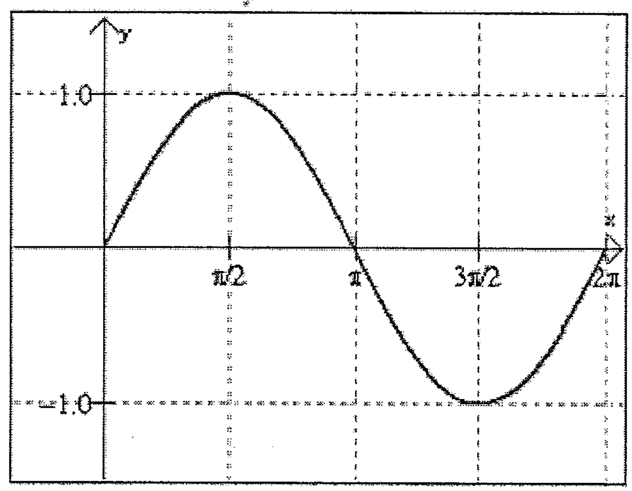
Points you should know from the unit circle

| x | Angle in Radians | 0 | π/2 | π | 3π/2 | 2π |
|---|---|---|---|---|---|---|
| y | sin x | | | | | |

### Sine Function: $y = \sin x$

- called a "wave" because of its rolling wave-like appearance (also referred to as oscillating)
- amplitude: 1 (height)
- period: $2\pi$ (length of one cycle)
- frequency: 1 cycle in $2\pi$ radians [or $1/(2\pi)$]
- domain: $\{x \mid x \in \mathbb{R}\}$
- range: $\{y \mid -1 \le y \le 1\}$

$y = \sin x$



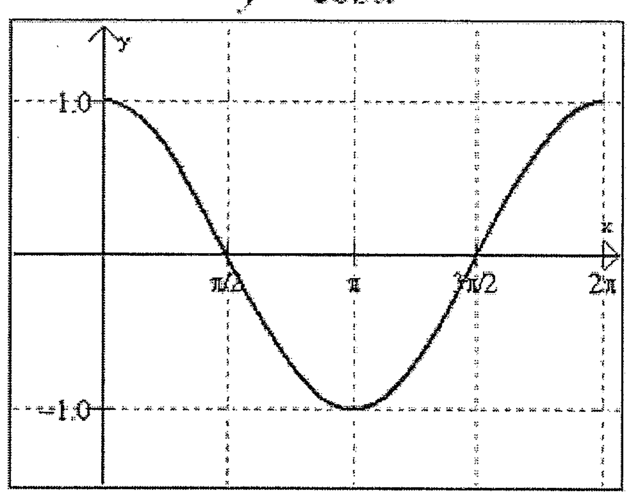

At $x = 0$, the sine wave is on the shoreline!
(meaning the y-value is equal to zero)

| x | Angle in Radians | 0 | π/2 | π | 3π/2 | 2π |
|---|---|---|---|---|---|---|
| y | cos x | | | | | |

### Cosine Function: $y = \cos x$

- called a "wave" because of its rolling wave-like appearance
- amplitude: 1
- period: $2\pi$
- frequency: 1 cycle in $2\pi$ radians [or $1/(2\pi)$]
- domain: $\{x \mid x \in \mathbb{R}\}$
- range: $\{y \mid -1 \le y \le 1\}$

$y = \cos x$



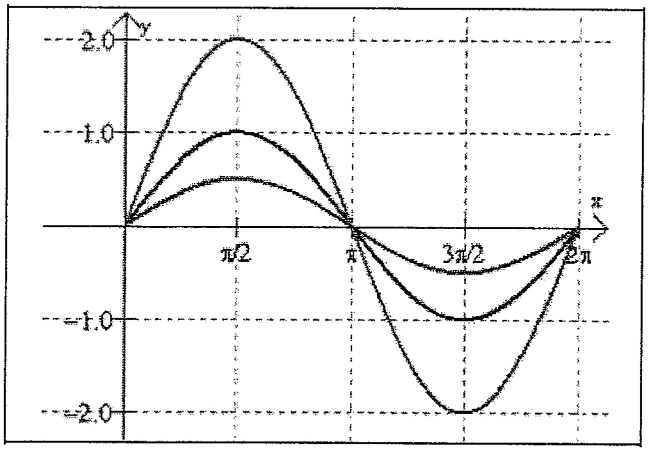

At $x = 0$, the cosine wave breaks off the cliff!
(meaning the y-value is equal to one)

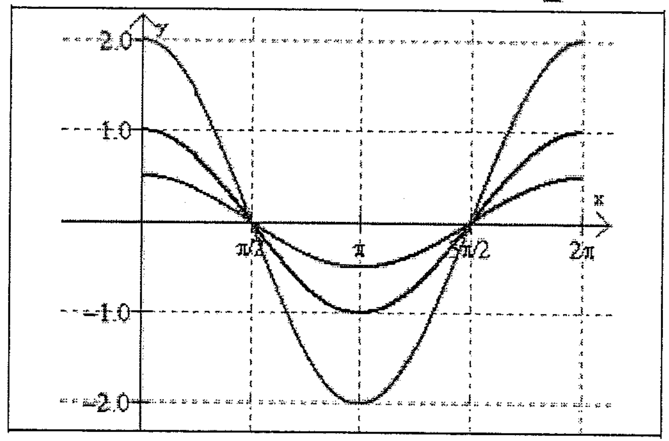
## The sine and cosine curves

$$y = A\sin(B(x - C)) + D \;\; and \; y = A\cos(B(x - C)) + D$$

The value **A** affects the amplitude. The amplitude (half the distance from the max to the min) will be |A| because distance is always ________________. Increasing or decreasing an A value with vertically ___________ or _______________ a graph.

$y = \sin x \quad y = 2\sin x \quad y = \frac{1}{2}\sin x \qquad y = \cos x \quad y = 2\cos x \quad y = \frac{1}{2}\cos x$

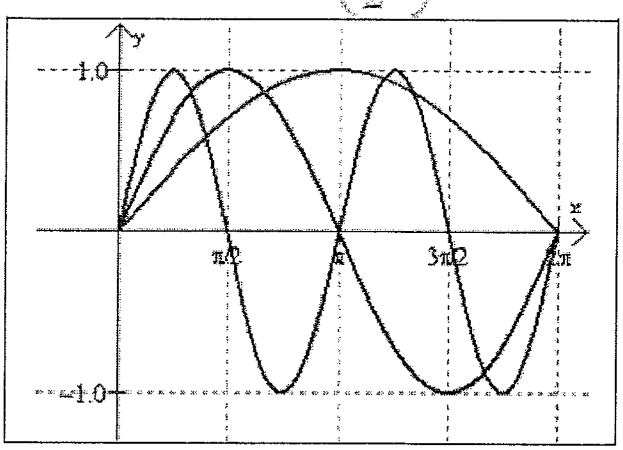


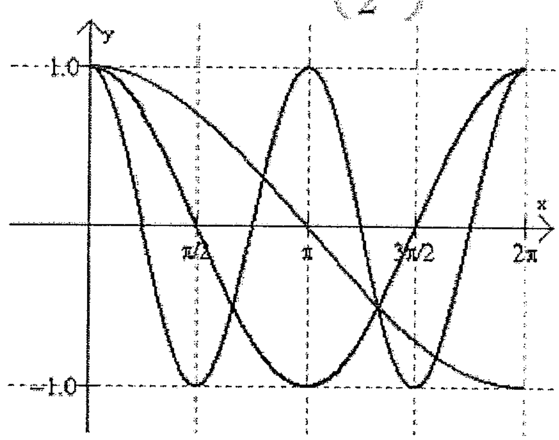



*The change in amplitude changes the "height" but not the width. This graph still reaches from 0 to 2π.*

The **B** value is the number of cycles it completes in an interval of _____________ or ______. The **B** value affects the period. The period of sine and cosine is $|\frac{2\pi}{b}|$. When 0<B<1 the period of the function is ____________ than 2π and the graph will have a ________________ ________________. When **B**>1, the period is ______________ than 2π and the graph will have a horizontal ________________.

$y = \sin x \quad y = \sin\left(\frac{1}{2}x\right) \quad y = \sin(2x) \qquad y = \cos x \quad y = \cos\left(\frac{1}{2}x\right) \quad y = \cos(2x)$

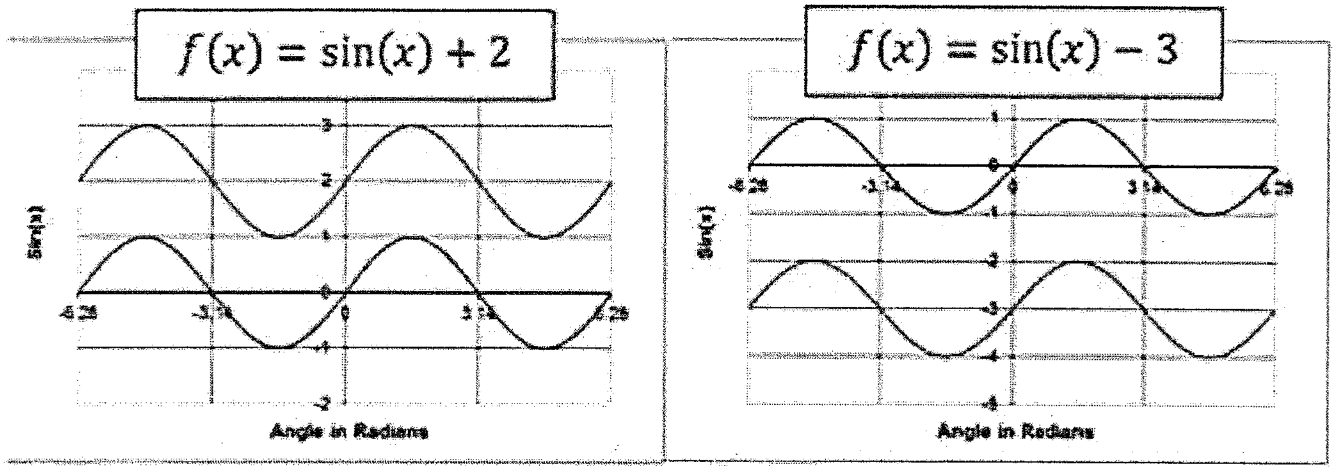





These graphs change horizontal "width" but do not change height. The two red graphs only show us half of the original graphs in their 0 to $2\pi$ windows. We would need to "stretch" the domain window to $4\pi$ to see entire cycles of those two graphs. The two blue graphs show us two complete cycles of the graphs in their 0 to $2\pi$ windows, which would allow us to "shrink" the domain window and still see complete cycles of the graphs.

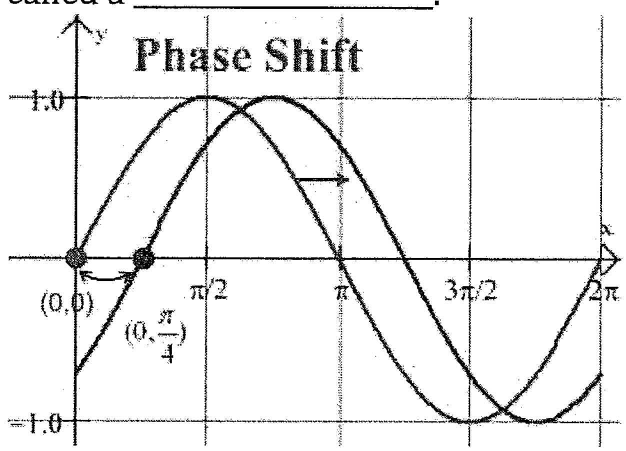
**3-9 Notes**

Just like any other function, adding a constant **on the end of the function** will shift the trig graph ________________ (up if the constant is ____________, down if the constant is ____________).




*To determine the midline of a graph you can add the max and min and divide by two.*

Just like any other function, adding a constant on **into the function** will shift the trig graph ________________ (left if the constant is ____________, right if the constant is ____________). This is called a ________________.




*Note: You may have to factor out B in order to determine the phase shift.*

**Graphing Trig Functions**

$$y = A\sin(B(x - C)) + D$$

$$y = A\cos(B(x - C)) + D$$

$|A|$ = amplitude

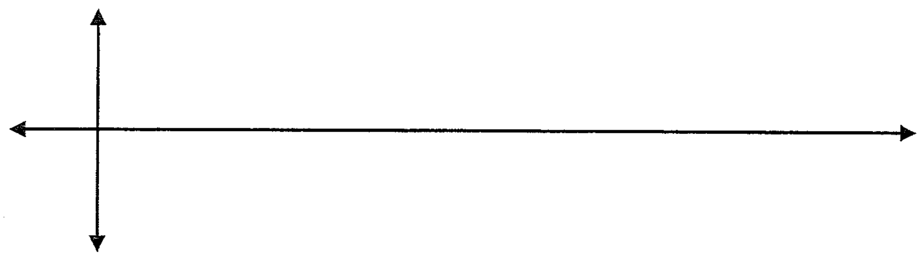
B = Horizontal Stretch, use to find period $|\frac{2\pi}{B}|$

C= Phase Shift (Horizontal Shift)

D= Vertical Shift (or "displacement")

**Sinusoidal Applications:** Using your knowledge of sine and cosine curves, you will be able to write the equation of applications using circular patterns.

**Example 1:** A weight is suspended from a spring. Assuming no friction or air resistance, when the weight is pulled down a small distance, it will oscillate indefinitely about the equilibrium position. If the weight is pulled down 3 cm then after 1 second it will be back at the equilibrium position, at 2 seconds it will be at the 3 cm above the equilibrium position, and 3 seconds it will be back at equilibrium and at 4 seconds and it will be 3cm below.



a) Find the equation of a sinusoidal function that will model this movement.

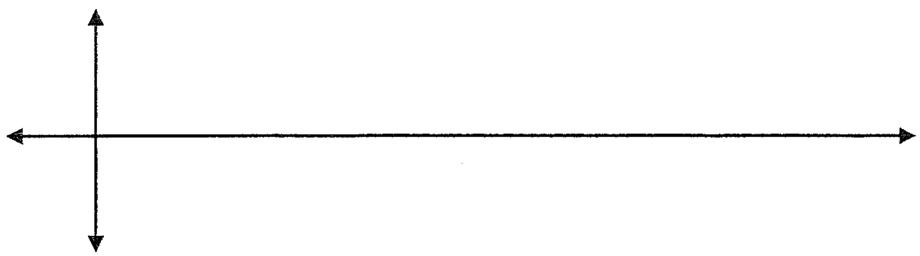
| | **Amplitude** (height from equilibrium; A=1/2 (Max-Min) and is it a reflection? | **Period** P=how long for one cycle, $\frac{2\pi}{b}$=P; solve for b | **Phase Shift** Did it move left/right? How much? | **Vertical Shift** Did it move up or down from the x-axis? How much? |
|---|---|---|---|---|
| **Sine** | | | | |
| **Cosine** | | | | |

Sine: ______________________ Cosine: ______________________

b) Find the distance of the weight from its equilibrium position, 1.5 seconds after release and 15 seconds after release?

**Example 2:** Tarzan is swinging back and forth on a grapevine. As he swings, he goes back and forth across riverbank, going alternately over land and water. Jane decides to model his movement mathematically and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the number of meters Tarzan is from the riverbank. Assume that y varies sinusoidally with t and that y is positive when Tarzan is over water and negative when he is over land.

Jane finds that when t=2, Tarzan is at the end of his swing where y=-23. She finds that when t is 5, he reaches the other end of his swing and y is 17.



a) Find the equation expressing Tarzan's distance from the riverbank in terms of t.

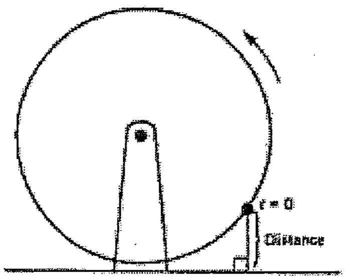
| | **Amplitude** (height from equilibrium; A=1/2 (Max-Min) and is it a reflection? | **Period** P=how long for one cycle, $\frac{2\pi}{b}$=P; solve for b | **Phase Shift** Did it move left/right? How much? | **Vertical Shift** Did it move up or down from the x-axis? How much? |
|---|---|---|---|---|
| **Sine** | | | | |
| **Cosine** | | | | |

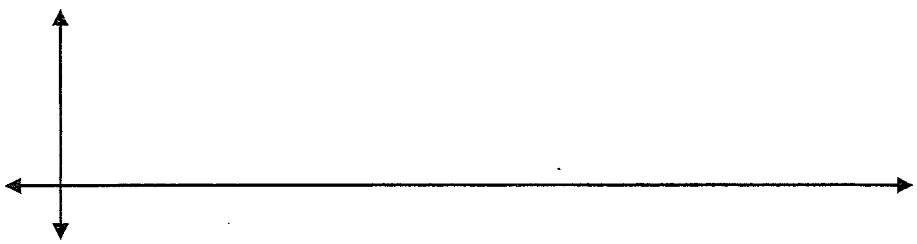
Sine: ______________________________ Cosine: ________________________________

b) Find y when t=2.8 and t=6.3

c) Where was Tarzan when Jane started the stopwatch?

**Example 3:** You've probably noticed that as you ride a Ferris wheel, your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts, your seat is at the position shown in the diagram below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 feet.





a) What is the lowest you go as the Ferris wheel turns, and why is this number greater than zero?

Find the equation expressing Tarzan's distance from the riverbank in terms of t.

| | **Amplitude** (height from equilibrium; A=1/2 (Max-Min) and is it a reflection? | **Period** P=how long for one cycle, $\frac{2\pi}{b}$=P; solve for b | **Phase Shift** Did it move left/right? How much? | **Vertical Shift** Did it move up or down from the x-axis? How much? |
|---|---|---|---|---|
| **Sine** | | | | |
| **Cosine** | | | | |

Sine: ____________________ Cosine: ____________________

c) Predict the height above the ground when t=9.