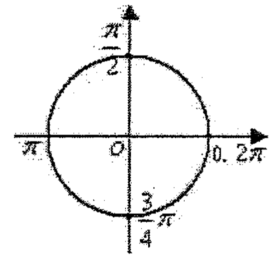
Angle and Radian Measure

Measurements of Angles: Until now you have measured angles in degrees. Another unit for measuring angles is radians.

Radians: the angle made by taking the radius of a circle and wrapping it around the edge.

-The circumference of a circle with radius 1 is 2π so a complete revolution has made 2π radians.

-A straight angle (or $\frac{1}{2}$ of a circle) has measure π radians.



Converting Radians and Degrees:

$$\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times \text{degrees}$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times \text{radians}$$

Examples:

1. Express 60° in radians $\frac{60^\circ}{1} \times \frac{\pi}{180} = \frac{60\pi}{180} = \frac{1\pi}{3} = \frac{\pi}{3}$

2. Express $\frac{\pi}{6}$ rad in degrees $\frac{\cancel{\pi}}{6} \cdot \frac{180}{\cancel{\pi}} = \frac{180}{6} = 30^\circ$

Practice:

#1-8, change the given angle to radians.

1) 315° $\frac{315^\circ}{1} \cdot \frac{\pi}{180} = \frac{315\pi}{180}$ $\frac{7\pi}{4}$

2) -60° $\frac{-60^\circ}{1} \cdot \frac{\pi}{180} = \frac{-60\pi}{180}$ $-\frac{\pi}{3}$

3) 212° $\frac{212^\circ}{1} \cdot \frac{\pi}{180} = \frac{212\pi}{180}$ $= 53\pi/45$

4) -168° $\frac{-14\pi}{15}$

5) 12.5° $\frac{5\pi}{72}$

6) -310° $\frac{-31\pi}{18}$

7) 600° $10\pi/3$

8) -720° -4π

#9-16, change the given angle to degrees.

9) $\frac{3\cancel{\pi}}{4} \cdot \frac{180}{\cancel{\pi}} = \frac{540}{4}$ $= 135^\circ$

10) $-\frac{9\pi}{5} \cdot \frac{180}{\pi} = -324^\circ$

11) $\frac{15\pi}{8}$ 337.5°

12) $-\frac{\pi}{10}$ -18°

13) $\frac{7\pi}{10}$ 126°

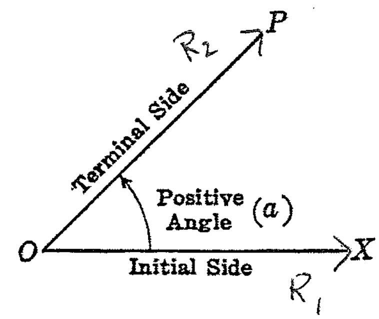
14) $-\frac{16\pi}{15}$ 192°

15) $\frac{88\pi}{9}$ 1760°

16) $-\frac{29\pi}{12}$ -435°

Angles in Standard Position

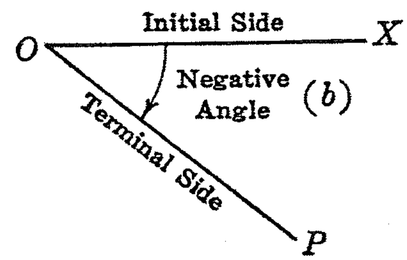
Angle: Consists of 2 rays, R_1 and R_2 with a common vertex O



Initial Side: The side in which the rotation of the angle begins, R_1

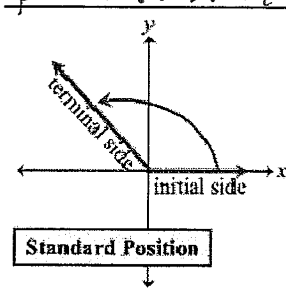
Terminal Side: The side which rotates to form the angle R_2

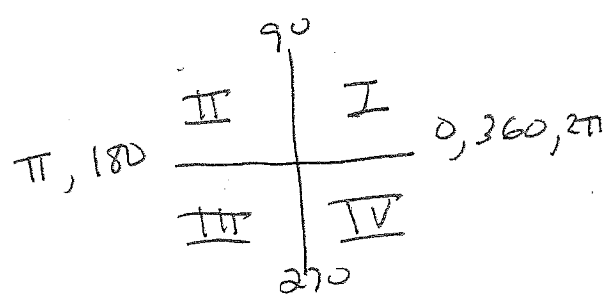
Positive Angle: when the rotation of an angle occurs counterclockwise



Negative Angle: when the rotation of the angle occurs clockwise

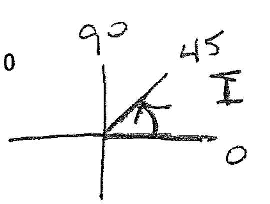
An angle is in standard position if it is draw in the xy-plane with its vertex at the origin and its initial side on the positive x axis.



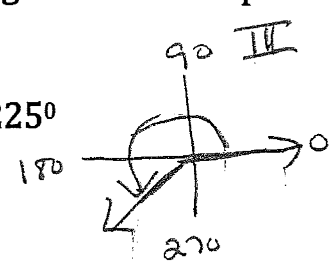


Example: Draw the given angle in standard position. State the quadrant the terminal side is in.

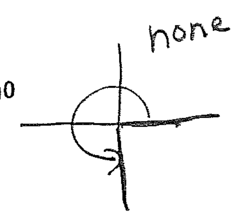
1. 45^0



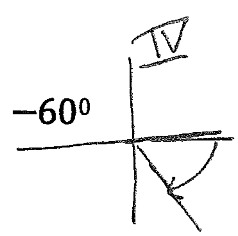
2. 225^0



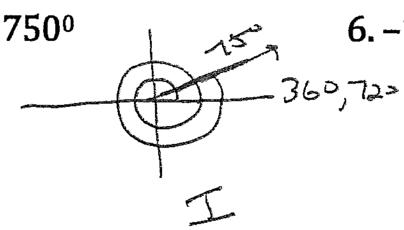
3. 270^0



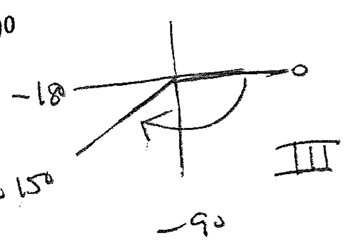
4. -60^0



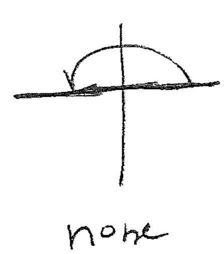
5. 750^0



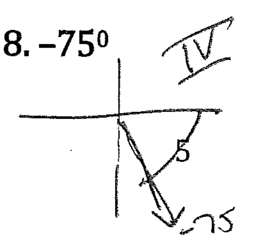
6. -150^0



7. 180^0



8. -75^0



Graphs of Sine and Cosine

From your discovery activity yesterday, you should have discovered that sine and cosine values repeat themselves. Thus, the sine and cosine functions are periodic.

A function is periodic if there is a positive number p such that f(t + p) = f(t) for every t. The least such positive number is called the period.

Points you should know from the unit circle

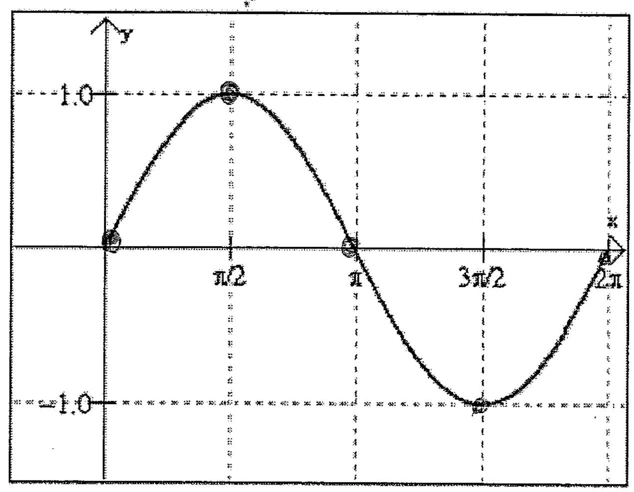
x	Angle in Radians	0	π/2	π	3π/2	2π
y	sin x	0	1	0	-1	0

Sine Function: $y = \sin x$

- called a "wave" because of its rolling wave-like appearance (also referred to as oscillating)
- amplitude: 1 (height)
- period: 2π (length of one cycle)
- frequency: 1 cycle in 2π radians [or $1/(2\pi)$]
- domain: $\{x \mid x \in \mathbb{R}\}$
- range: $\{y \mid -1 \le y \le 1\}$

soil

$y = \sin x$



At $x = 0$, the sine wave is on the shoreline!
(meaning the y-value is equal to zero)

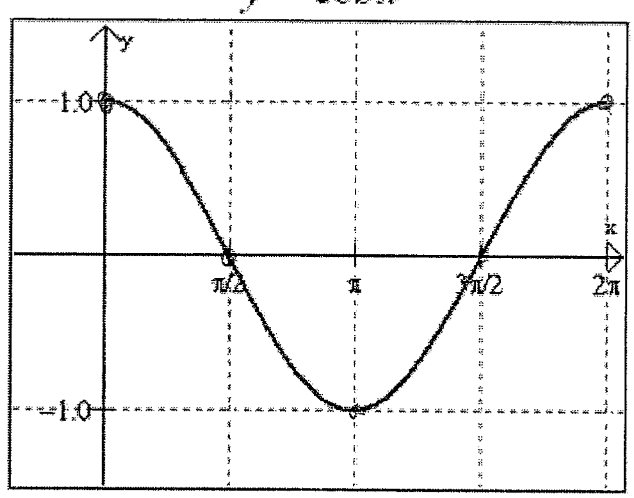
x	Angle in Radians	0	π/2	π	3π/2	2π
y	cos x	1	0	-1	0	1

Cosine Function: $y = \cos x$

- called a "wave" because of its rolling wave-like appearance
- amplitude: 1
- period: 2π
- frequency: 1 cycle in 2π radians [or $1/(2\pi)$]
- domain: $\{x \mid x \in \mathbb{R}\}$
- range: $\{y \mid -1 \le y \le 1\}$

clouds

$y = \cos x$



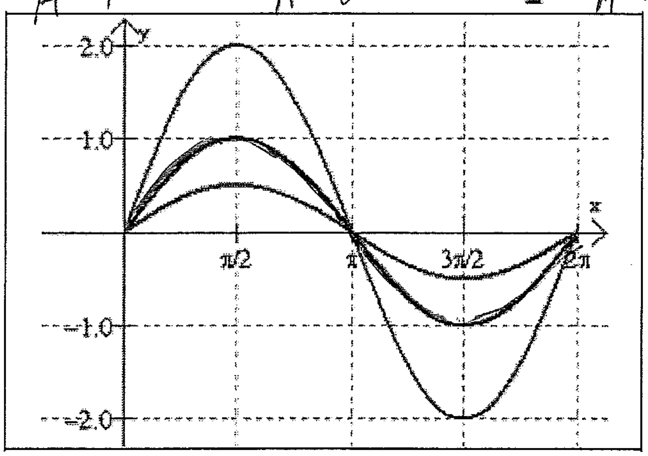
At $x = 0$, the cosine wave breaks off the cliff!
(meaning the y-value is equal to one)

The sine and cosine curves

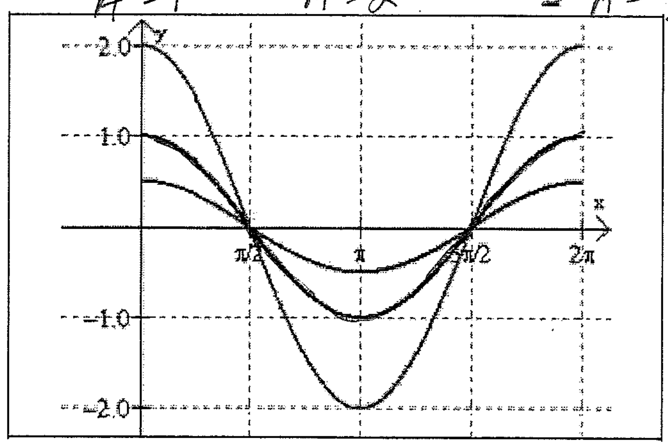
$y = A\sin(B(x - C)) + D$ *and* $y = A\cos(B(x - C)) + D$

The value **A** affects the amplitude. The amplitude (half the distance from the max to the min) will be |A| because distance is always positive. Increasing or decreasing an A value with vertically stretch or shrink a graph.

$y = \sin x$ $A=1$ $y = 2\sin x$ $A=2$ $y = \frac{1}{2}\sin x$ $A=\frac{1}{2}$



$y = \cos x$ $A=1$ $y = 2\cos x$ $A=2$ $y = \frac{1}{2}\cos x$ $A=\frac{1}{2}$

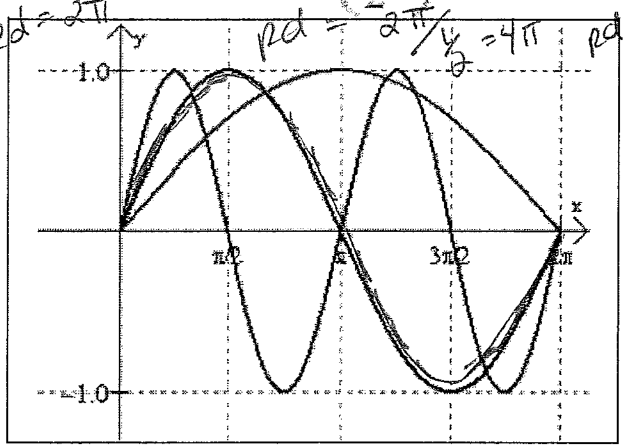


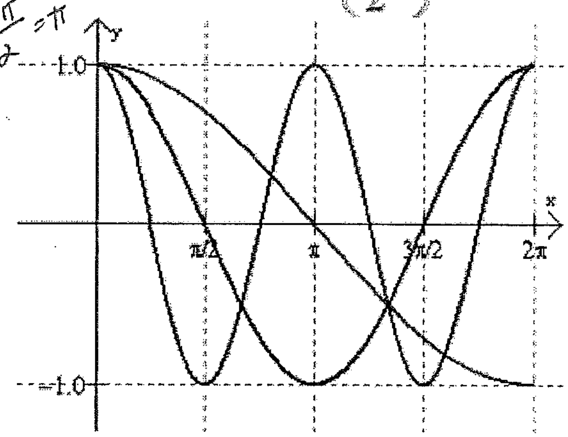
The change in amplitude changes the "height" but not the width. This graph still reaches from 0 to 2π.

The **B** value is the number of cycles it completes in an interval of 360° or 2π. The **B** value affects the period. The period of sine and cosine is $|\frac{2\pi}{b}|$. When 0<B<1 the period of the function is greater than 2π and the graph will have a horizontal stretch. When **B**>1, the period is less than 2π and the graph will have a horizontal Shrink.

$y = \sin x$ $\quad y = \sin\left(\frac{1}{2}x\right)$ $\quad y = \sin(2x)$ $\quad y = \cos x$ $\quad y = \cos\left(\frac{1}{2}x\right)$ $\quad y = \cos(2x)$

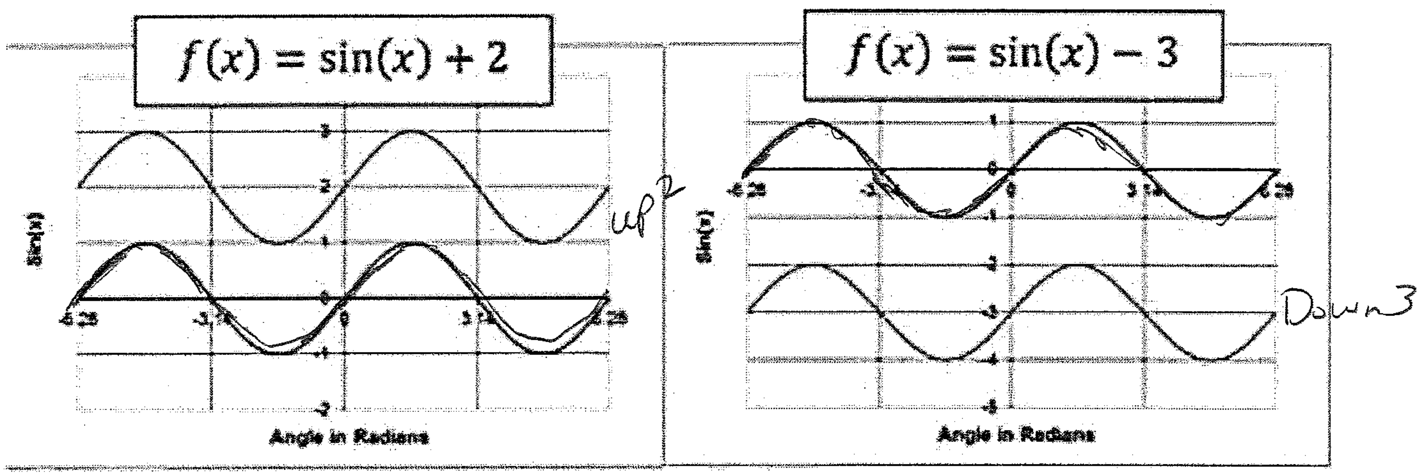
$pd = 2\pi$ $\quad pd = 2\pi/\frac{1}{2} = 4\pi$ $\quad pd = \frac{2\pi}{2} = \pi$





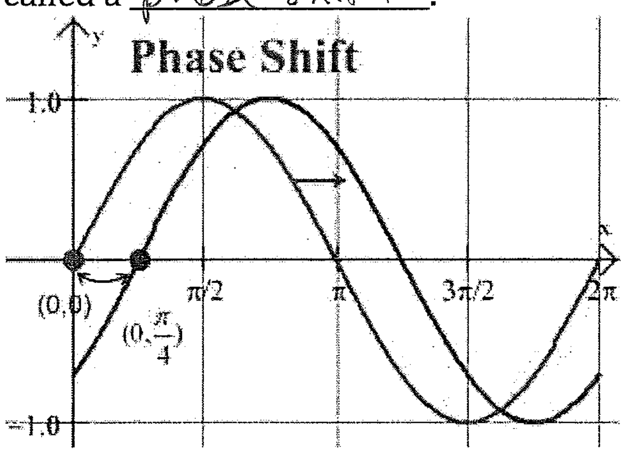
These graphs change horizontal "width" but do not change height. The two red graphs only show us half of the original graphs in their 0 to 2π windows. We would need to "stretch" the domain window to 4π to see entire cycles of those two graphs. The two blue graphs show us two complete cycles of the graphs in their 0 to 2π windows, which would allow us to "shrink" the domain window and still see complete cycles of the graphs.

Just like any other function, adding a constant **on the end of the function** will shift the trig graph vertically (up if the constant is positive, down if the constant is negative).



To determine the midline of a graph you can add the max and min and divide by two.

Just like any other function, adding a constant on **into the function** will shift the trig graph horizontally (left if the constant is positive, right if the constant is negative). This is called a phase shift.



Note: You may have to factor out B in order to determine the phase shift.

Graphing Trig Functions

$y = \cancel{A\sin(B(x - C)) + D}$ → $y = A\sin(Bx + C) + D$

$y = \cancel{A\cos(B(x - C)) + D}$ $y = A\cos(Bx + C) + D$

$|A|$ = amplitude

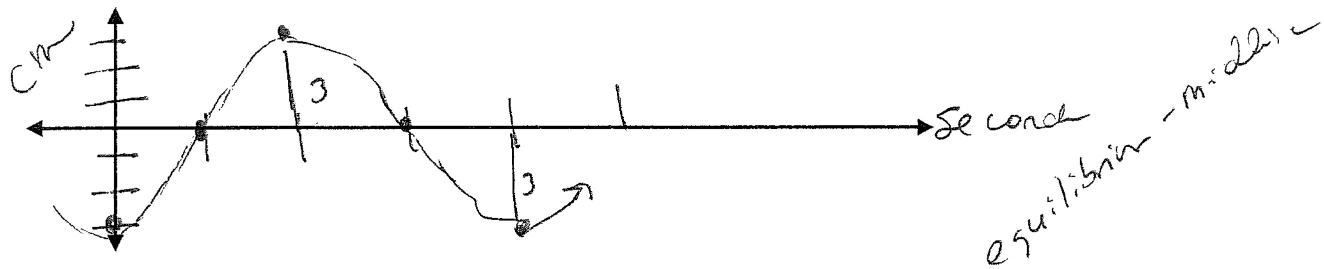
B = Horizontal Stretch, use to find period $|\frac{2\pi}{B}|$

~~C=~~ Phase Shift (Horizontal Shift) $-C/B$

D= Vertical Shift (or "displacement")

Sinusoidal Applications: Using your knowledge of sine and cosine curves, you will be able to write the equation of applications using circular patterns.

Example 1: A weight is suspended from a spring. Assuming no friction or air resistance, when the weight is pulled down a small distance, it will oscillate indefinitely about the equilibrium position. If the weight is pulled down 3 cm then after 1 second it will be back at the equilibrium position, at 2 seconds it will be at the 3 cm above the equilibrium position, and 3 seconds it will be back at equilibrium and at 4 seconds and it will be 3cm below.



a) Find the equation of a sinusoidal function that will model this movement.

	Amplitude (height from equilibrium; A=1/2 (Max-Min) and is it a reflection?	Period P=how long for one cycle, $\frac{2\pi}{b}$=P; solve for b	Phase Shift Did it move left/right? How much?	Vertical Shift Did it move up or down from the x-axis? How much?
Sine	Amp = 3 * no reflection - going up	Period = 4 sec $\frac{4}{1} = \frac{2\pi}{b}$ $b = \frac{\pi}{2}$	sine starts one to the right Right 1	none midline at 0
Cosine	$\frac{1}{2}$(max-min) $= \frac{1}{2}(3 - -3)$ $= \frac{1}{2}(6)$ $= 3$ * reflection - going down	Period = 4 seconds $\frac{4}{1} = \frac{2\pi}{b}$ $4b = 2\pi$ $b = 2\pi/4 = \frac{\pi}{2}$	none started on y axis	none midline is at 0

Sine: $y = 3\sin(\frac{\pi}{2}(x-1))$ Cosine: $y = -3\cos(\frac{\pi}{2}x)$

$y = A\cos(bx) + d$

b) Find the distance of the weight from its equilibrium position, 1.5 seconds after release and 15 seconds after release?

* radian mode for sinusoidal application
* can use either equation

@1.5 sec → 2.12 above equil.
@15 sec → 0 above equil.

y =
$-3\cos((\frac{\pi}{2})\cdot(1.5)) = 2.12$ above equilibrium

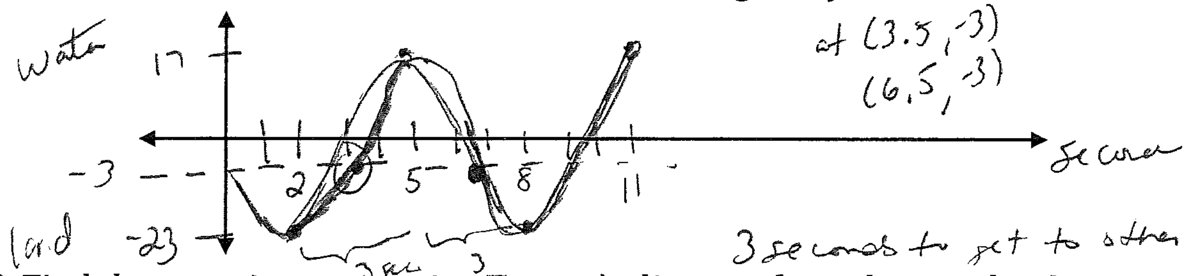
$3\sin((\frac{\pi}{2})(1.5-1)) = 2.12$ above equilibr

Example 2: Tarzan is swinging back and forth on a grapevine. As he swings, he goes back and forth across riverbank, going alternately over land and water. Jane decides to model his movement mathematically and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the number of meters Tarzan is from the riverbank. Assume that y varies sinusoidally with t and that y is positive when Tarzan is over water and negative when he is over land.

Jane finds that when t=2, Tarzan is at the end of his swing where y=-23. She finds that when t is 5, he reaches the other end of his swing and y is 17.

midline - halfway between

$\frac{max+min}{2} = \frac{17+-23}{2} = \frac{-6}{2} = -3$



3 seconds to get to other end

a) Find the equation expressing Tarzan's distance from the riverbank in terms of t.

start at midline halfway between points of top & bottom (half of 3 is 1.5)

	Amplitude (height from equilibrium; A=1/2 (Max-Min) and is it a reflection?	**Period** P=how long for one cycle, $\frac{2\pi}{b}$=P; solve for b	**Phase Shift** Did it move left/right? How much?	**Vertical Shift** Did it move up or down from the x-axis? How much?
Sine ~~reflection~~	distance from middle is 20 or $\frac{1}{2}$(max-min) $\frac{1}{2}(17--23)=20$	$\pi/3$	3.5 to the right	Down 3 $y=-3$
Cosine	20 reflection	Period = 2 to 8 which is 6 second	Right 2	Where is midline? Down 3 $y=-3$

sine starts at 3.5

$\frac{6}{1} = \frac{2\pi}{b}$ $6=2\pi$ $b=\pi/3$

Sine: $y = 20 \sin\left(\pi/3(x-3.5)\right)-3$ Cosine: $y = -20\cos\left(\pi/3(x-2)\right)-3$

r2 d3

b) Find y when t=2.8 and t=6.3

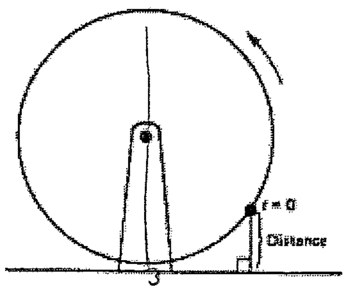
$y = 20\sin(\pi/3(2.8-3.5))-3 = -16.38$

$y = 20\sin(\pi/3(6.3-3.5))-3 = 1.16$

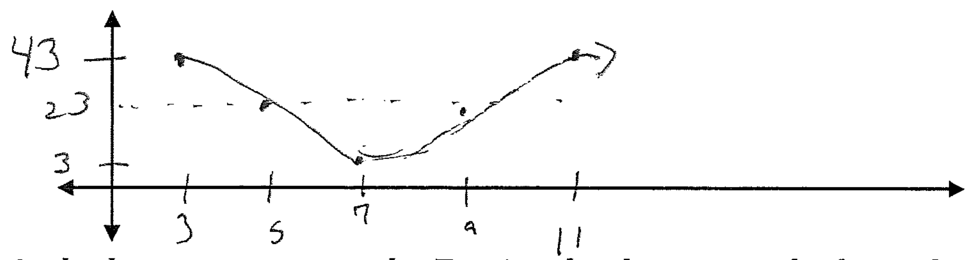
c) Where was Tarzan when Jane started the stopwatch?

$y = 20\sin(\pi/3(0-3.5))-3 = 7$

Example 3: You've probably noticed that as you ride a Ferris wheel, your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts, your seat is at the position shown in the diagram below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 feet.



midline = $\frac{1}{2}$ (max + min)
$= \frac{1}{2}(43 + 3)$
$= \frac{1}{2}(46)$
$= 23$



a) What is the lowest you go as the Ferris wheel turns, and why is this number greater than zero? 3 because the ferris wheel is 3 ft above the ground

Find the equation expressing Tarzan's distance from the riverbank in terms of t.

	Amplitude (height from equilibrium; A=1/2 (Max-Min) and is it a reflection?	**Period** P=how long for one cycle, $\frac{2\pi}{b}$=P; solve for b	**Phase Shift** Did it move left/right? How much?	**Vertical Shift** Did it move up or down from the x-axis? How much?
Sine				
Cosine	amp $= \frac{1}{2}$(max − min) $= \frac{1}{2}(43-3)$ $= \frac{1}{2}(40)$ $= 20$	8 sec. $\frac{2\pi}{b} = 8$ $2\pi = 8b$ $b = \frac{\pi}{4}$	moved R 3	midline is up 23

Sine: ______________________ Cosine: ______________________

* north.

$y = 20\cos\left(\frac{\pi}{4}(x-3)\right) + 23$

c) Predict the height above the ground when t=9.