Day 1: Things to recall from Geometry about right triangles:

- The angles of a triangle add up to 180 degrees.
- One angle is 90 degrees, therefore leaving the other angles to be acute (less than 90 degrees). Why? Because if the angles add up to 180 and one is 90, the other two have to be less than 90 together or otherwise it would be greater than 180 degrees.
- The side across from the right angle is called the hypotenuse. This is always the longest side.
- θ = theta, the measure of an angle (could be degrees or radians but we will use degrees for right now).
- If you know 2 sides of a right triangle, you can find the other because of the Pythagorean Theorem. $a^2 + b^2 = c^2$ (Whatever it equals squared must be the hypotenuse. It is NOT necessarily side c each time in each triangle, it just depends on how it is labeled)
- Each angle is represented with a capital letter and its corresponding side is represented by the lower case letter of that.

Pythagorean Theorem

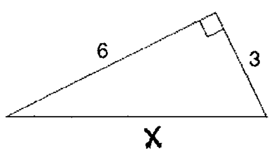
a. Pythagorean Theorem is used to find missing sides in a triangle.

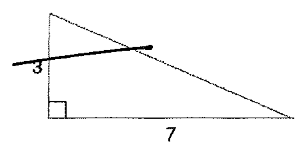
$$\boxed{a^2 + b^2 = c^2 \qquad leg^2 + leg^2 = hyp^2}$$

b. "a" and "b" represent the legs

c. "c" represents the hypotenuse

d. Examples: Find the missing sides using Pythagorean Theorem

i. 

2. 

$6^2 + 3^2 = x^2$

$36 + 9 = x^2$

$45 = x^2$

$\sqrt{45} = x$

$3\sqrt{5} = x$

$6.71 \approx x$

45
9 5
3 3

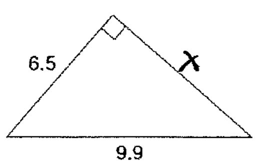
$\cancel{6.5^2 + 9.9^2 =}$

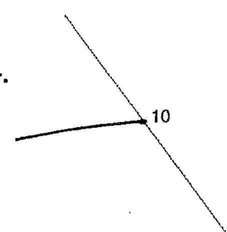
$6.5^2 + x^2 = 9.9^2$

$x^2 = 9.9^2 - 6.5^2$

$x^2 = 55.7600$

$x \approx 7.47$

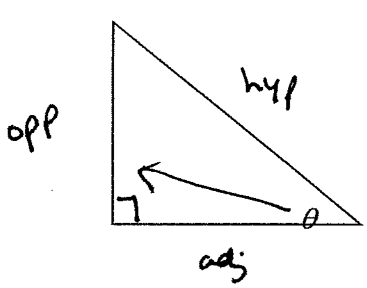
3. 

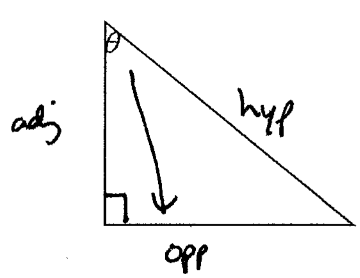
4. 

Labeling Triangles

In a triangle, depending on where theta is, depends on where the side that is opposite or adjacent is. Opposite – straight across from (opposite of it), and adjacent – right next to.

Ex:



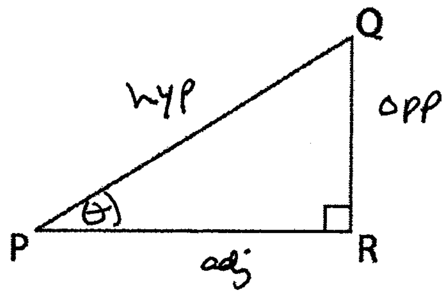


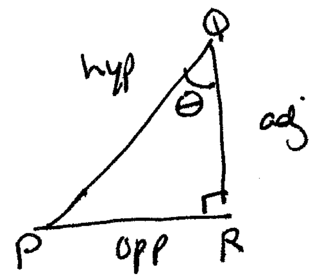
Notes 3-1, Trigonometric Ratios

Name ____________________
Date __________ Period __________

Trigonometric Ratios in Right Triangles

- The trig ratios are just the <u>ratios</u> formed by the lengths of the sides of a right triangle.
- There are <u>6</u> trig ratios, though we will only be working with <u>3</u>.





∠P:
- Opposite leg to ∠P: $\overrightarrow{QR}$
- Adjacent leg to ∠P: $\overrightarrow{PR}$
- Hypotenuse: $\overrightarrow{PQ}$

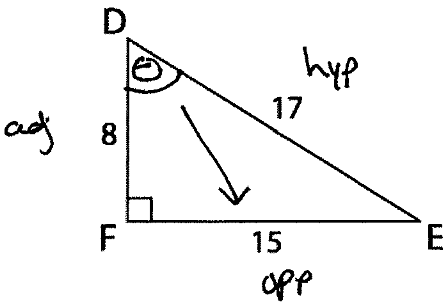
∠Q:
- Opposite leg to ∠Q: $\overrightarrow{PR}$
- Adjacent leg to ∠Q: $\overrightarrow{QR}$
- Hypotenuse: $\overrightarrow{PQ}$

Trigonometric Ratio	Abbreviation	Definition
Sine of ∠P	Sin P	$\frac{\text{opposite leg to } \angle P}{\text{hypotenuse}} = \frac{O}{H} = \underline{\sin P}$
Cosine of ∠P	Cos P	$\frac{\text{adjacent leg to } \angle P}{\text{hypotenuse}} = \frac{A}{H} = \underline{\cos P}$
Tangent of ∠P	Tan P	$\frac{\text{opposite leg to } \angle P}{\text{adjacent leg to } \angle P} = \frac{O}{A} = \underline{\tan P}$

Setting up Trigonometry Ratios and Solving for Sides

ii. <u>locate the angle</u> (NOT the right angle)
iii. <u>label the sides</u> (Opposite, Adjacent, Hypotenuse)
iv. <u>Choose the ratio</u>:
- ✓ <u>sin</u> if we have the opposite and hypotenuse
- ✓ <u>cos</u> if we have the adjacent and the hypotenuse
- ✓ <u>tan</u> if we have the opposite and the adjacent

Ex 1: Given the following right triangle, write the trig ratios in fraction and decimal form.



Sin D = $15/17$ = .8824

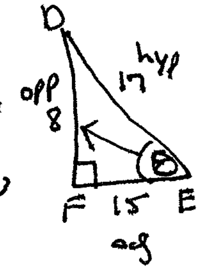
Cos D = $8/17$ = .4706

Tan D = $15/8$ = 1.875

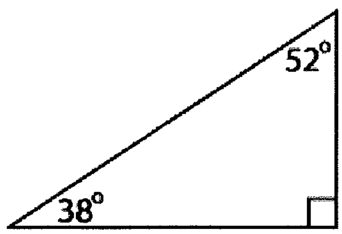
Sin E = $8/17$ = .4706

Cos E = $15/17$ = .8824

Tan E = $8/15$ = .5333



Ex 2: Find the following trig values using the calculator. (Make sure your calculator is in degree mode and then round to four decimal places)



Sin 38° = .6157

Cos 38° = .7880

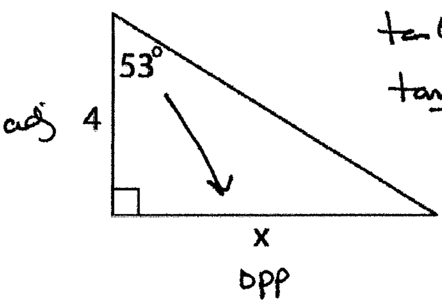
Tan 38° = .7813

Sin 52° = .7880

Cos 52° = .6157

Tan 52° = 1.280

Ex 3: Find the value of x.



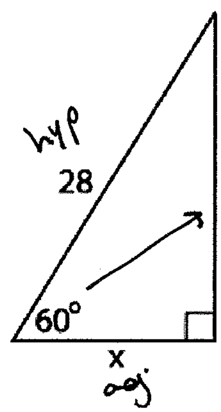
$\tan\theta = o/a$

$\frac{\tan 53°}{1} = \frac{x}{4}$

$x = (\tan 53°)(4)$

$x \approx 5.3082$

Ex 4: Find the value of x.



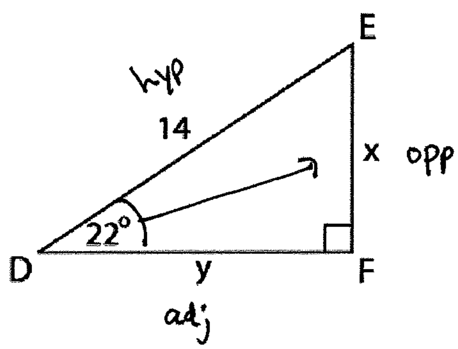
$\cos\theta = a/h$

$\frac{\cos 60°}{1} = \frac{x}{28}$

$x = (\cos 60°)(28)$

$x \approx 14$

Ex 5: Find the values of x and y.



$\frac{\sin 22°}{1} = \frac{x}{14}$

$x = (\sin 22°)(14)$

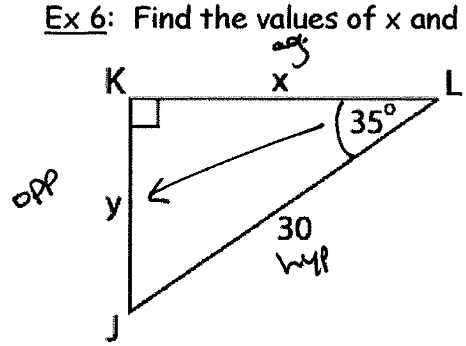
$x \approx 5.2445$

$\frac{\cos 22°}{1} = \frac{y}{14}$

$y = (\cos 22°)(14)$

$y \approx 12.9806$

Ex 6: Find the values of x and y.



$\frac{\sin 35°}{1} = \frac{y}{30}$

$y = (\sin 35°)(30)$

$y \approx 17.2073$

$\frac{\cos 35°}{1} = \frac{x}{30}$

$x = (\cos 35°)(30)$

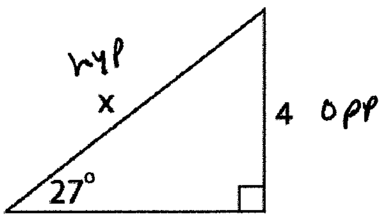
$x \approx 24.5746$

Notes 3-2, Inverse Trigonometric Functions

Name ______________________
Date ___________ Period __________

Inverse Trig Functions

When solving a problem in which you're looking for the value of a side of a triangle,
you set up the problem using a trig ratio
and you solve the problem using a proportion



$\sin\theta = o/h$

$\frac{\sin 27°}{1} = \frac{4}{x}$

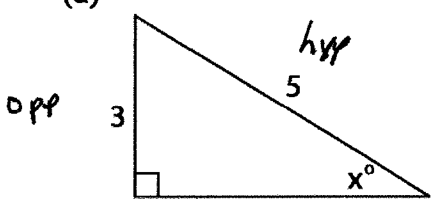
$(\sin 27°)(x) = 4$

$x = 4/\sin 27°$ $\quad x \approx 8.81$

When solving a problem in which you're looking for the value of an angle of a triangle
you set up the problem using a trig ratio,
but you solve the problem using an inverse trig ratio.

Ex 1: Find the value of x.

(a)



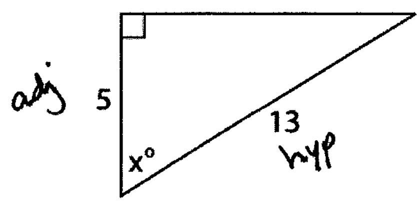
$\sin\theta = o/h$

$\sin x = 3/5$

$x = \sin^{-1}(3/5)$

$x \approx 37°$

(b)



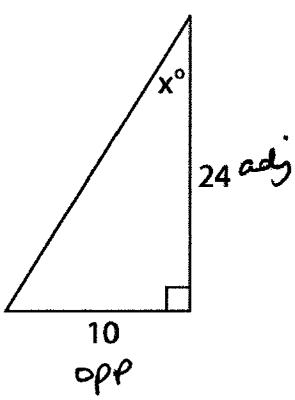
$\cos\theta = a/h$

$\cos x = 5/13$

$x = \cos^{-1}(5/13)$

$x \approx 67°$

(c)



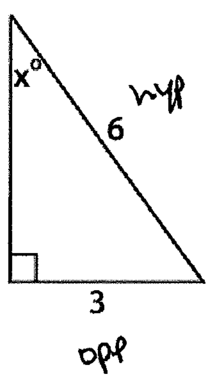
$\tan\theta = o/a$

$\tan x = 10/24$

$x = \tan^{-1}(10/24)$

$x \approx 23°$

(d)



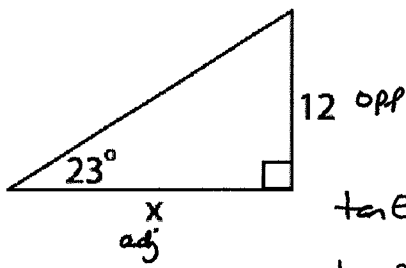
$\sin\theta = o/h$

$\sin x = 3/6$

$x = \sin^{-1}(3/6)$

$x \approx 30°$

Ex 2: Missing Side



$\tan\theta = o/a$

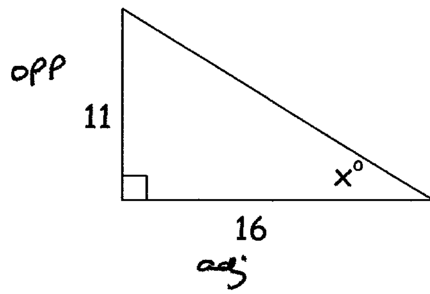
$\frac{\tan 23°}{1} = \frac{12}{x}$

$(\tan 23°)(x) = 12$

$x = \frac{12}{\tan 23°}$

$x \approx 28.3$

Ex 3: Missing angle



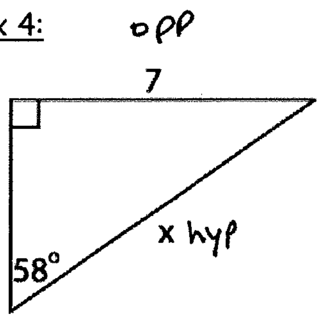
$\tan\theta = o/a$

$\tan x = 11/16$

$x = \tan^{-1}(11/16)$

$x \approx 34.5°$

Ex 4:



$\sin\theta = o/h$

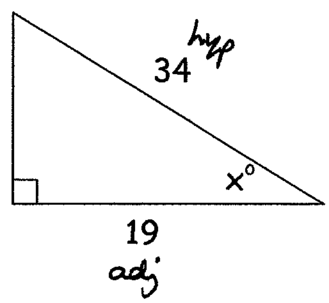
$\frac{\sin 58°}{1} = \frac{7}{x}$

$(\sin 58°)(x) = 7$

$x = 7/\sin 58°$

$x \approx 8.3$

Ex 5:



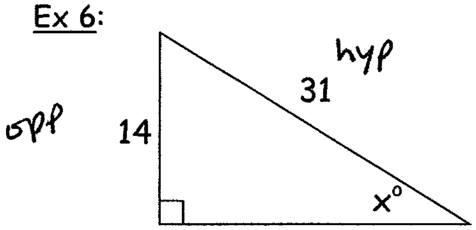
$\cos\theta = a/h$

$\cos x = 19/34$

$x = \cos^{-1}(19/34)$

$x \approx 56°$

Ex 6:



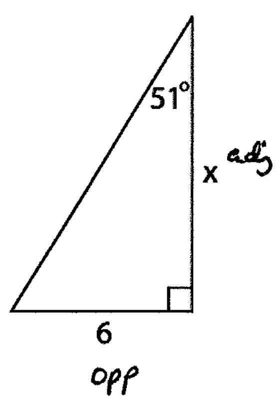
$\sin\theta = o/h$

$\sin x = 14/31$

$x = \sin^{-1}(14/31)$

$x \approx 26.8°$

Ex 7:



$\tan\theta = o/a$

$\frac{\tan 51°}{1} = \frac{6}{x}$

$(\tan 51°)(x) = 6$

$x = \frac{6}{\tan 51°}$

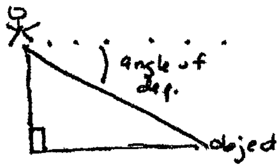
$x \approx 4.9$

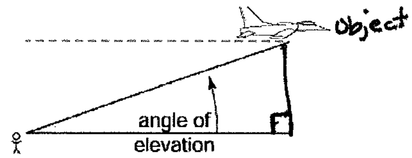
Notes 3-3, Angles of Elevation and Depression

Name ____________________
Date ____________ Period ____________

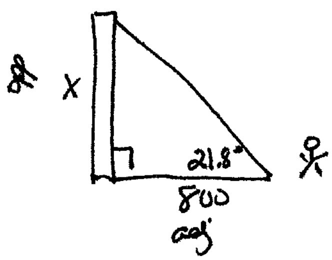
Angles of Elevation and Depression

Angle of Elevation = angle of depression





Ex 1: A person is standing 800 m away from the base (bottom) of the Chrysler building in New York. The angle of elevation to look up to the top of the building is 21.8°. How tall is the building?



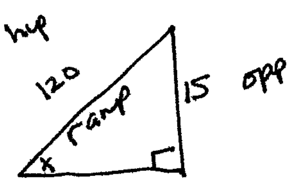
$\tan\theta = o/a$

$\frac{\tan 21.8°}{1} = \frac{x}{800}$

$x = (\tan 21.8°)(800)$

$x \approx 320$ meters

Ex 2: A ramp is 120 ft long and rises vertically 15 ft. Find the angle of elevation of the ramp.



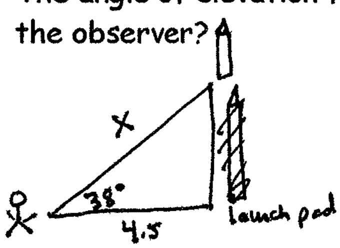
$\sin\theta = o/h$

$\sin x = 15/120$

$x = \sin^{-1}(15/120)$

$x \approx 7.2°$

Ex 3: A mission control observer, 4.5 km from the launch pad, observes a space shuttle ascending. The angle of elevation from the observer to the shuttle is 38°. How far is the shuttle from the observer?



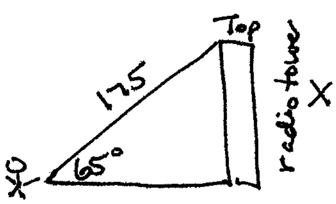
$\cos\theta = a/h$

$\frac{\cos 38°}{1} = \frac{4.5}{x}$

$(\cos 38°)(x) = 4.5$

$x = 4.5/\cos 38°$ $x \approx 5.7$ km

Ex 4: The length of a wire supporting a radio tower is 175 ft. The angle of elevation to the top of the radio tower from the foot of the wire is 65°. How tall is the radio tower?



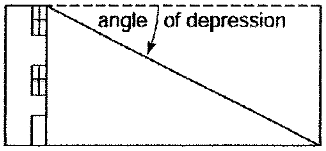
$\sin\theta = o/h$

$\frac{\sin 65°}{1} = \frac{x}{175}$

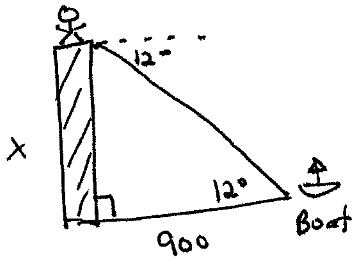
$x = (\sin 65°)(175)$

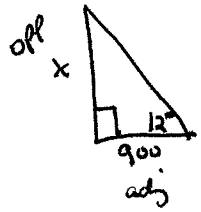
$x \approx 158.6$ feet

Angle of Depression -



Ex 5: From the top of a lighthouse, an observer notices a boat and finds the angle of depression to be 12°. If the boat is 900 ft from the bottom of the lighthouse, what is the height of the lighthouse?





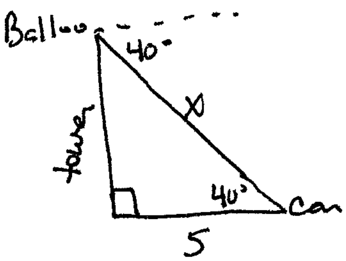
$\tan\theta = o/a$

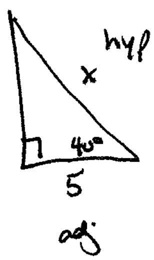
$\frac{\tan 12°}{1} = \frac{x}{900}$

$x = (\tan 12°)(900)$

$x \approx 191.3$ ft

Ex 6: The angle of depression of a car from a hot air balloon basket is 40°. The balloon is directly over a water tower that is 5 km from the car. Find the direct distance from the balloon to the car.





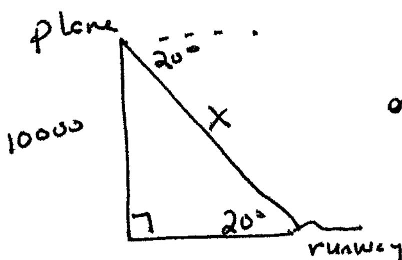
$\frac{\cos 40°}{1} = \frac{5}{x}$

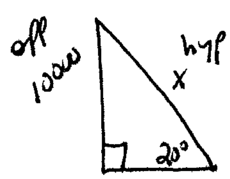
$(\cos 40°)(x) = 5$

$x = 5/\cos 40°$

$x \approx 6.5$ km

Ex 7: A pilot is required to approach a runway at an angle of descent (depression) measuring 20°. If the altitude of the plane is 10,000 ft, what is the flight distance to the runway?





$\sin\theta = o/h$

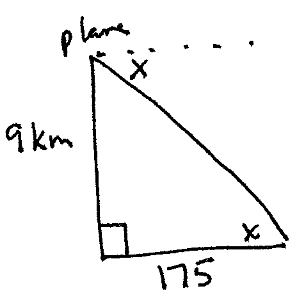
$\frac{\sin 20°}{1} = \frac{10000}{x}$

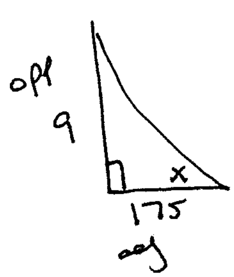
$(\sin 20)(x) = 10000$

$x = 10000/\sin 20$

$x \approx 29238$ feet

Ex 8: After flying at an altitude of 9 km, an airplane starts to descend when its ground distance from the landing field is 175 km. What is the angle of depression for this portion of the flight?





$\tan\theta = o/a$

$\tan x = 9/175$

$x = \tan^{-1}(9/175)$

$x \approx 2.9°$

Notes 3-4

Law of Sines

Law of sines suggests that in ANY triangle (not just right triangles) the lengths of the sides are proportional to the sines of the corresponding opposite angles.

Formula:

$$\frac{\text{Big Letters (Angles)}}{\text{Little Letters (sides)}} = \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

All are equal! It doesn't matter which ones you use!

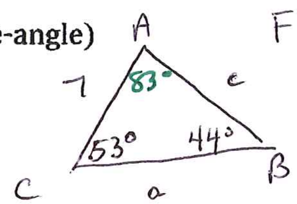
Use the law of sines if you are given a triangle with:

Side-Angle-Angle (SAA)

Side-Side-Angle (SSA)-

Examples

1. SAA (Side-angle-angle)



Find AB.

Find the third <.

$180° - 53° - 44° = \underline{83°}$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

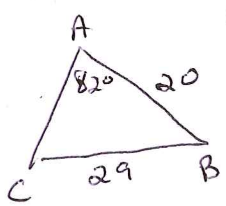
$$\frac{\sin 83}{a} = \boxed{\frac{\sin 44}{7} = \frac{\sin 53}{c}}$$

$(\sin 44)(c) = 7 \sin 53$

$c = \frac{7 \sin 53}{\sin 44}$

$c = 8$

2. SSA



Find m< C

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 82}{29} = \cancel{\frac{\sin B}{b}} = \frac{\sin C}{20}$$

$29 \sin C = (\sin 82)(20)$

$\sin C = \frac{(\sin 82)(20)}{29}$

$\sin C = .6829$

$C = \sin^{-1}(.6829)$

$C \approx 43.1°$

4. A satellite orbiting the earth passes directly overhead at observation stations in Phoenix and Los Angeles, 340 miles apart. At an instant when the satellite is between these two stations, its angle of elevation is simultaneously observed to be 60^0 at Phoenix and 75^0 at Los Angeles. How far is the satellite from Los Angeles

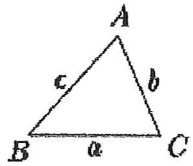
Notes 3-5

Law of Cosines

Law of Cosines is used when you are given the following information:

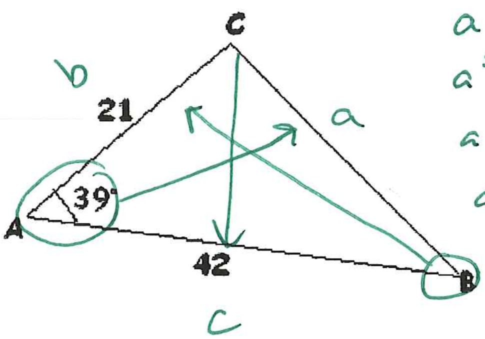
SSS- Know all three sides and no angles

SAS- Know 2 sides and the angle between them



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = c^2 + a^2 - 2ca\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Example 1: In ΔABC, m∠A=39, AC=21 and AB=42. Find side a to the nearest integer.



$a^2 = b^2 + c^2 - 2bc\cos A$

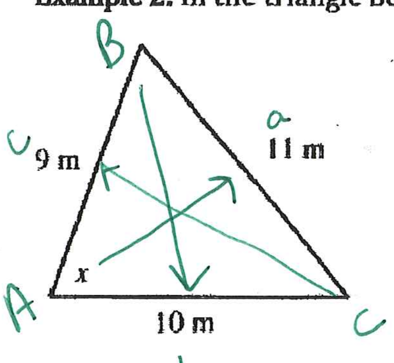
$a^2 = 21^2 + 42^2 - 2(21)(42)\cos 39°$

$a^2 = 834.1145$

$a = \sqrt{834.1145}$

$a \approx 28.9$

Example 2: In the triangle below, find the measure of angle x.



1 term

$a^2 = b^2 + c^2 - \boxed{2bc\cos A}$

$11^2 = 10^2 + 9^2 - \boxed{2(10)(9)\cos A}$

$121 = 100 + 81 - \boxed{180\cos A}$

$121 = 181 - \boxed{180\cos A}$

$-181 \quad -181$

$-60 = \boxed{-180\cos A}$

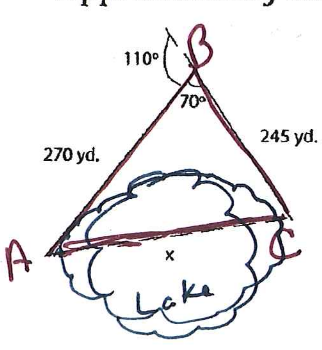
$\frac{-60}{-180} = \frac{-180\cos A}{-180}$

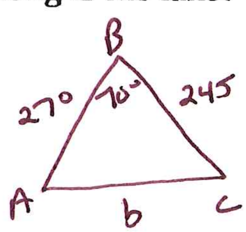
$.3333 = \cos A$

$A = \cos^{-1}(.3333)$

$A \approx 71°$

To approximate the length of a lake, a surveyor starts at one end of the lake and walks 245 yards. He then turns 110º and walks 270 yards until he arrives at the other end of the lake. Approximately how long is the lake?





$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 270^2 + 245^2 - 2(270)(245)\cos 70°$

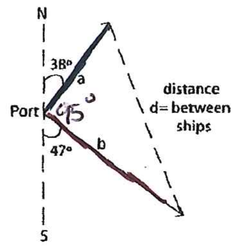
$b^2 = 87675.7350$

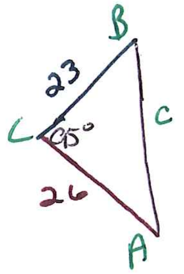
$b = \sqrt{87675.7350}$

$b \approx 296$ yds

Two ships leave port at 4 p.m. One is headed at a bearing of N 38 E and is traveling at 11.5 miles per hour. The other is traveling 13 miles per hour at a bearing of S 47 E. How far apart are they when dinner is served at 6 p.m.?

traveled for 2 hrs





$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 23^2 + 26^2 - 2(23)(26)\cos 95$

$c^2 = 1309.2383$

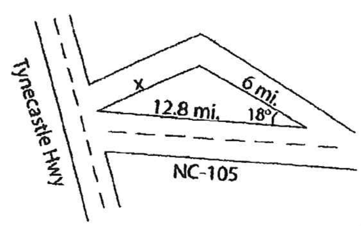
$c = \sqrt{1309.2383}$

$c \approx 36$ miles

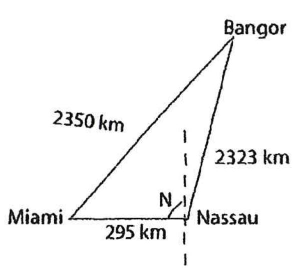
2hrs @ 11.5 mph = 23 miles

2hrs @ 13 mph = 26 miles

You are heading to Beech Mountain for a ski trip. Unfortunately, state road 105 in North Carolina is blocked off due to a chemical spill. You have to get to Tynecastle Highway which leads to the resort at which you are staying. NC-105 would get you to Tynecastle Hwy in 12.8 miles. The detour begins with a 18 veer off onto a road that runs through the local city. After 6 miles, there is another turn that leads to Tynecastle Hwy. Assuming that both roads on the detour are straight, how many extra miles are you traveling to reach your destination?



The distance on a map from the airport in Miami, FL to the one in Nassau, Bahamas is 295 kilometers due east. Bangor, Maine is northeast of both cities; its airport is 2350 kilometers from Miami and 2323 kilometers from Nassau. What bearing would a plane need to take to fly from Nassau to Bangor?



After the hurricane, the small tree in my neighbor's yard was leaning. To keep it from falling, we nailed a 6-foot strap into the ground 4 feet from the base of the tree. We attached the strap to the tree 3½ feet above the ground. How far from vertical was the tree leaning?