Tree Diagrams and the Fundamental Counting Principle

The purpose of this task is to help students discover the Fundamental Counting Principle through the use of tree diagrams. Additionally, the tree diagrams are used to solve problems related to cost and probability.

As an introduction to this activity, situations could be given to small groups for them to prepare tree diagrams and then compare the number of possible outcomes for each situation. Small groups could share their tree diagrams and the outcomes. By listing the situations and the possible outcomes students should be able to determine the relationship between the number of decisions and the number of outcomes.

Situations for Group Practice:
Assign one of the following situations to each group. Give each group a piece of paper and marker. Each group should create a tree diagram to determine the number of outcomes.

1. Flip a dime and then flip a quarter
2. A choice of chicken, fish or beef for the main dish and a choice of cake or pudding for dessert
3. A choice of either a green or blue shirt and a choice of blue, black or khaki pants
4. A choice of pizza or spaghetti; a choice of milk or juice to drink; a choice of pudding or an apple for dessert
5. Shirts come on three sizes: small, medium or large; shirts have buttons or snaps; colors are blue or beige
6. The choices for school mascot are lion, bear and porpoise; colors are red, blue and gold

As students present their work create a table like the one below:

Decisions	Possible Outcomes
2, 2	4
3, 2	6
2, 4	8
2, 2, 2	8
3, 3	9
3, 2, 2	12

Ask: Is there a relationship between the number of decisions to be made and the possible outcomes? *The possible outcomes is the product of the decisions – this is the* ***Fundamental Counting Principle***

The **Fundamental Counting Principle** tells us that if we have two decisions to make, and there are ***M*** ways to make the first decision, and ***N*** ways to make the second decision, the product of ***M*** and ***N*** tells us how many different outcomes there are for the overall decision process. In general, when a series of decision are to be made, the product of all the way to make the individual decisions determines the number of outcomes there are.

The following pages offer two scenarios for students to solve probability problems using tree diagrams.

AFM Notes, Unit 1 – Probability Name______________________

1-1 FPC and Permutations Date____________ Period ____

The Fundamental Principle of Counting

How many different outfits could you put together using two sweaters, four pairs of pants, and two pairs of shoes?

The **Fundamental Principle of Counting** says: Suppose there are ______ ways of choosing one item, and ______ ways of choosing a second item, and ______ ways of choosing a third item, and so on. Then the total number of possible outcomes is ____________________.

The probability of an event is: P(Event) =

Ex 1) Suppose a license plate can have any three letters followed by any four digits.

a) How many different license plates are possible?

b) How many license plates are possible that have no repeated letters or digits?

c) What is the **probability** that a randomly selected license plate has no repeated letters or digits?

Permutations

Ex. 2) I have five books I want to arrange (in a particular order) on a shelf.

a) How many different ways can I arrange them?

b) What if I only want to arrange 3 of my 5 books on a shelf? How many ways can I do this?

Whenever you want to know how many ways there are of ________________________ some number of items, that's called a ________________________________.

With **Permutations**
____
____

Ex 3: Seven flute players are performing in an ensemble.

a) The names of all seven players are listed in the program in random order. How many different ways could the players' names be listed (i.e., **arranged)** in the program?

b) How many different ways could the players' names be listed in alphabetical order by last name?

c) If the players' names are listed in the program in random order, what is the **probability** that the names happen to be in alphabetical order?

d) After the performance, the players are backstage. There is a bench with only room for four to sit. How many possible **arrangements** are there for four of the seven players to sit on the bench?

AFM Notes, Unit 1 – Probability
1-2 Combinations

Name__________________________
Date____________ Period _______

<u>Permutations vs. Combinations</u> (Electing Officers vs. Forming a Committee)

Ex. 1) We want to elect three officers from our club of 25 people. The first person elected will be the President, the second person elected will be the Vice President, and the third person elected will be the Treasurer. How many different "arrangements" of officers can we have?

Ex. 2) We want to form a 3-person committee (i.e., no officers) from our club of 25 people. How many committees can we form?

When you're counting how many ways there are to ____________ some number of items, ___________ _____________; that's a ______________________.

When you're counting how many ways there are to simply ________ some number of items, __________ does _______ ____________; that's a ____________________.

Ex. 3) The Debate Club wants to elect four officers (Pres, VP, Sec, and Treas), from its membership of 30 people. How many different ways could the Debate Club elect its officers?

Ex. 4) The Debate Club wants to create a 4-person committee (i.e., no officers) from its membership of 30 people. How many different committees are possible?

Combinations with Restrictions

Ex. 5) The Young Republicans Club consists of 7 seniors, 9 juniors, and 5 sophomores. They want to form a Planning Committee (i.e., without officers) to plan their spring social. The Planning Committee will consist of 4 members.

a) How many different 4-member committees are possible?

b) How many committees are possible that consist of all sophomores?

c) How many different committees could be formed if the club's president must be one of the members?

d) How many different committees could be formed if the committee must contain exactly two seniors and two juniors?

AFM Notes, Unit 1 – Probability
1-3 Factorial Notation

Name___________________________
Date_____________ Period ________

Ex. 1) I have eight books I want to arrange on a shelf.

a) How many different ways can I arrange the eight books?

1) Using the permutations operation on the calculator
2) Using the Fundamental Principle of Counting

A third way to express this answer is by using ________________ notation:

b) What if I only want to arrange 3 of my 8 books on a shelf? How many ways can I do this?

Again, we've already discussed two ways to calculate the answer to this problem.

1) Using the permutations operation on the calculator
2) Using the Fundamental Principle of Counting

We can also express this answer by using factorial notation

This last expression is actually the formula for a permutation. If we want to calculate the number of permutations of ___ objects taken ____ at a time, we would write:

Ex. 2) Calculate the expression 120!/116!

Ex. 3) Calculate the expression 76!/73!

Ex. 4) Calculate the expression n!/(n-3)!

Ex. 5) Calculate the expression ${}_nP_{n-3}$

AFM Notes, Unit 1 - Probability Name ______________________________

1-4 Introduction to Probability Date _____________ Period ___________

Probability theory was initially developed in 1654 in a series of letters between two French mathematicians, Blaise Pascal and Pierre de Fermat, as a means of determining the fairness of games. It is still used today to make sure that casino customers lose more money than they win, and in many other areas, including setting insurance rates.

At the heart of probability theory is ________________. Rolling a die, flipping a coin, drawing a card and spinning a game board spinner are all examples of ___________________________. In a random process no individual event is predictable, even though the long range pattern of many individual events often is predictable.

Types of Probability

Experimental –

Theoretical –

Calculating Probabilities

When calculating the probability of something happening, the "something" is called an ________, and the probability of the event happening is written _______________.

Ex. 1a) The probability of rolling a 3 on a die would be written _________________________.
Ex. 1b) The probability of winning the lottery would be written ________________________.

Probabilities are always expressed as __________________________________. The probability of an event that is certain to happen is ___, while the probability of an impossible event is ___.

To calculate a probability, you count the ____
and divide this number by the total ____.

Probability of an event: P(E) =

Example of Theoretical Probability

Ex. 2) A bag contains 4 blue marbles, 6 green marbles and 3 yellow marbles. A marble is drawn at random from the bag.

a) What's the probability of drawing a green marble?

b) What's the probability of drawing a yellow marble?

c) What's the probability of drawing a green OR yellow marble?

Example of Experimental Probability

Ex. 3) Suppose a study of car accidents and drivers who use mobile phones produced the following data:

	Had a car accident in the last year	Did not have a car accident in the last year	Totals
Driver using mobile phone	45	280	325
Driver not using mobile phone	25	405	430
Totals	70	685	755

This type of table is called a ____

The total number of people in the sample is ______. The row totals are ______ and ______.

The column totals are ____ and _____. Notice that 325 + 430 = _____, and 70 + 685 = _____.

Calculate the following probabilities using the table above:

a) P(a driver is a mobile phone user) =

b) P(a driver had no accident in the last year) =

c) P(a driver using a mobile phone had no accident in the last year) =

AFM Notes, Unit 1 Probability **Name** ______________________________

1-5 More Probability **Date** ____________ **Period** ___________

A compound probability is a probability involving ________________________ events, for example, the probability of Event A _____ Event B happening.

Example 1: Coin Flip

What's the probability of flipping a coin twice and having it come up heads both times?

P(Comes up heads) =

P(1^{st} flip comes up heads AND 2^{nd} flip comes up heads)

The Multiplication Rule

When calculating the probability of two events, Event A and Event B, if the events are ___________________, then the probability of both events happening is ________________

Compound Probability and Replacement

Example 2: Marbles

A bag contains 4 blue marbles, 6 green marbles and 3 yellow marbles. If two marbles are drawn at random from the bag, what's the probability of:

a) First drawing a green marble, and then drawing a yellow marble?

With replacement Without replacement

b) Drawing two blue marbles?

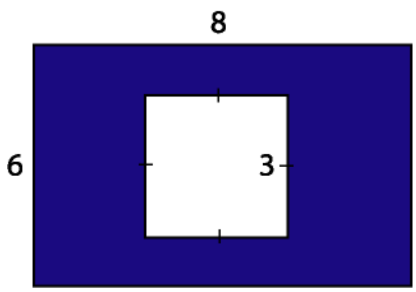
With replacement Without replacement

Geometric Probability - a probability that is found by calculating a _____________ of _____________ or _____________ of a geometric figure.

P(Event) =

Example 3: Geometric Probability of Rectangles

a) What is the probability that a point chosen at random in the rectangle will also be in the square.

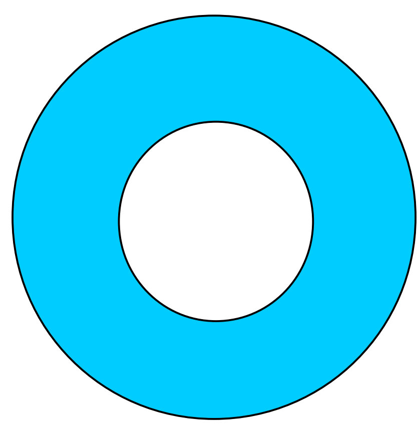


(b) What is the probability that a point chosen at random in the rectangle will be in the shaded area?

Example 4: Geometric Probability of Circles

The radius of the inner circle is 6cm, and the radius of the outer circle is 16cm. Find the probability that a point selected at random in the outer circle will be in the

(a) inner circle



(b) shaded area

Mutually Exclusive Events

When you roll a die, an event such as rolling a 1 is called a _______________ ____________. because it consists of only one event.

An event that consists of two or more simple events is called a ___________________ _______________. Such as the event of rolling an odd number or a number greater than 5.

_______________ ______________ _________________ is when two evens cannot occur at the same time. Like the probability of drawing a 2 or an ace is found by adding their individual probabilities.

If two events, A and B, are mutually exclusive, then the probability of A or B occurs is the sum of their probabilities.

P(A or B) = P(A) + P(B)

Example 5: Two Mutually Exclusive Events
Keisha ha a stack of 8 baseball cards, 5 basketball cards, and 6 soccer cards. If she selects a card at random from a stack, what is the probability that it is a baseball or a soccer card?

Example 6: Three Mutually Exclusive Events
There are 7 girls and 6 boys on the junior class homecoming committee. A subcommittee of 4 people is being chosen at random to decide the theme for the class float. What is the probability that the subcommittee will have a least 2 girls?

Inclusive Events

Since it is possible to draw a card that is both queen and a diamond, these events are not mutually exclusive, they are _______________ ______________.

If two events, A and B, are inclusive, then the probability that A or N occurs is the sum of their probabilities decreased by the probability of both occurring.

P(A or B) = P(A) + P(B) – P(A and B)

Example 7: Education
The enrollment at Southburg High school is 1400. Suppose 550 students take French, 700 take algebra, and 400 take both French and algebra. What is the probability that a student selected at random takes French or algebra?

Conditional Probability

The probability of an event under the condition that some preceding even has occurred is called ___________________ ________________. The conditional probability that event A occurs __________ that event B occurs can be represented by __________.

The conditional probability of event A, given event B, is defined as

Example 8: Medicine
Refer to the application below. What is the probability that a test subject's hair grew, given that he used the experimental drug?

	Number of Subjects	
	Using Drug	Using Placebo
Hair Growth	1600	1200
No Hair Growth	800	

AFM Notes, Unit 1 Probability Name ______________________________

1-6 Probability Distribution Date ____________ Period ___________

_____________________ _____________________ can be a table, graph, or equation that links each possible _________________ of an event with its probability of occurring.

- The probability of each outcome must be between ______ and ______.
- The sum of all the probabilities must equal _______.

Making a Probability Distribution

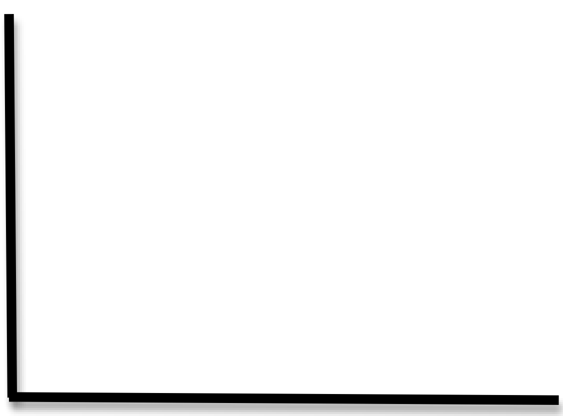
Example 1: Bakery

A bakery is trying a new recipe for the fudge deluxe cake. Customers were asked to rate the flavor of the cake on a scale of 1 to 5, with 1 being not tasty, 3 being okay, and 5 being delicious. Use the frequency distribution show to construct and graph a probability distribution.

Step 1: Find the proabability of each score.

Score, x	Frequency	P(x)
1	1	
2	8	
3	20	
4	16	
5	5	

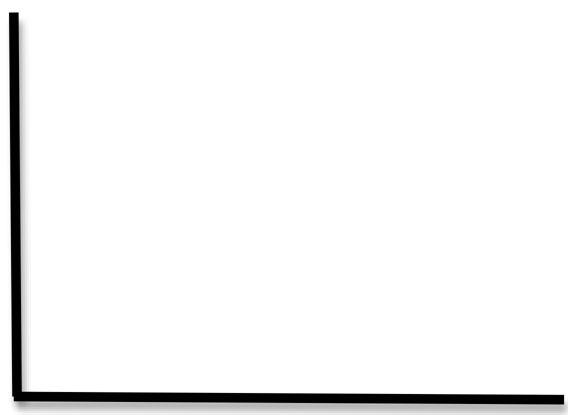
Step 2: Graph the score versus the probability.



Example 2: Car Sales

A car salesperson tracked the number of cars she sold each day during a 30-day period. Use the frequency distribution of the results to construct and graph a probability didstribution for the random variable x, rounding each proability to the nearest hundreth.

Cars Sold, x	Frequency	P(x)
0	20	
1	7	
2	2	
3	1	



Expected Value

In a random experiment, the values of the n outcomes are ______________________________ and the corresponding probabilities of the outcomes occurring are ____________________________.

The expected value (EV) of the experiment is given by:

____

To calculate expected value:

- Start with the ______________________ ______________________ or create it if you don't have it.
- Multiply the value of each ____________________ by it's ______________________.
- Add up all those products.
- The sum is the _____________________ ______________________.

Example 3: Fundraisers

At a raffle, 500 tickets are sold at $1 each for three prizes of $100, $50, and $10. What is the expected value of your net gain if you buy a ticket?

Gain, X	$100 – $1 or $99	$50 – $1 or $49	$10 - $1 or $9	$0 – 1 or -$1
Probability P(x)				

Example 4: Water Park

A water park makes $350,000 when the weather is normal and loses $80,000 per season when there are more bad weather days than normal. If the probability of having more bad weather days than normal this season is 35%, find the park's expected profit.

Gain, x		
Probability, P(x)		

Example 5: MP3 Players

Construct a probability distribution and find the expected value:

Students were asked how many MP3 players they own.

Players, x	Frequency	P(x)
0	9	
1	17	
2	9	
3	5	
4	2	

AFM Notes, Unit 1 Probability **Name** ______________________________

1-7 Binomial Theorem **Date** ____________ **Period** ___________

According to the U.S. Census Bureau, ten percent of families have three or more children. If a family has four children, there are six sequences of births of boys and girls that result in two boys and two girls. Find these sequences.

Pascal's Triangle

You can use the coefficients in powers of ______________________ to count the number of possible _______________ in situation such as the one above. Remember that a binomial is a polynomial with _______________ _______________. Expand a few powers of the binomial b + g.

The _____________________ 6 of the b^2g^2 in the expansion of $(b + g)^4$ gives the number of sequences of births that result in two boys and two girls. As another example the coefficient 4 of the b^1g^3 term gives the number of sequences with one boy and three girls.

Here are some patterns that can be seen in any binomial expansion of the form $(a + b)^n$.

1. There are ________________ terms.
2. The exponent n of ___________________ is the exponent of *a* in the first term and the exponent of *b* in the last term.
3. In successive terms, the exponent of *a* _________________ by one, and the exponent of *b* __________________ by one.
4. The sum of the exponents in each term is _______.
5. The coefficients are _________________. They ___________________ at the beginning of the expansion and ____________________ at the end.

The coefficients form a pattern that is often displayed in a triangular formation. This is known as ___________________ ________________. Notice that each row begins and ends with 1. Each coefficient is the sum of the two coefficients about t in the previous row

$(a + b)^0$						1					
$(a + b)^1$					1		1				
$(a + b)^2$				1		2		1			
$(a + b)^3$			1		3		3		1		
$(a + b)^4$		1		4		6		4		1	
$(a + b)^5$	1		5		10		10		5		1

Example 1: Use Pascal's Triangle
Expand $(x + y)^7$
Write two more rows of Pascal's triangle.

Use the patterns of a binomial expression and the coefficients to write the expansion of $(x+ y)^7$

$(x + y)^7 =$

The Binomial Theorem

Another way to show the coefficients in a binomial expansion is to write them in terms of the previous coefficients.

$(a+b)^0$					1				
$(a+b)^1$				1		__			
$(a+b)^2$			1		__		__		
$(a+b)^3$		1		__		__		__	
$(a+b)^4$	1		__		__		__		__

This pattern provides the ___________________ of $(a + b)^n$ for any nonnegative integer n. The pattern is summarized in the ____________________ ____________________.

If n is a nonnegative integer, then

$(a + b)^n =$

Example 2: Use the Binomial Theorem

Expand $(a - b)^6$

The expansion will have ______ terms. Use the sequence _______________________________ to find the coefficients for the first four terms. Then use ____________________ to find the remaining coefficients.

$(a - b)^6 =$

Note:

The factors in the coefficients of binomial expansion involve special products called ___________________. For example, the product of _________________ is written 4! And is read as ______________________. In general, if n is positive integer, then ____.

Example 3: Factorials

Evaluate $\frac{8!}{3!5!}$

An expression such as _______________ in Example 2 can be written as a quotient of factorials. In this case ________________. Using this idea, you can rewrite the expansion of $(a + b)^6$ using factorials:

You can also write this series using sigma notation.

In general, the ________________ ________________ can be written both in ______________ notation and in _______________ notation.

Example 4: Use a Factorial Form of the Binomial Theorem

Sometimes you need to know only a particular term of a binomial expansion. Note that when the Binomial Theorem is written in sigma notation, k = 0 for the first time, k = 1 for the second term, and so on. In general, the value of k is always one less than the number of term you are finding.

Example 5: Find a Particular Term
Find the fifth term in the expansion of $(p + q)^{10}$.
First, use the Binomial Theorem to write the expansion in sigma notation.

If the fifth term, k = 4.

AFM Notes, Unit 1 Probability Name ______________________________

1-8 Binomial Probability Date ____________ Period ___________

Conditions of a Binomial Experiment:

A ___________________ ___________________ exists if and only if these conditions occur.

- Each trial has exactly two _________________, or outcomes that can be reduced to two outcomes.
- There must be a _________________ number of trials.
- The out comes of the trial must be ___________________________.
- The probabilities in each trial are the __________________.

Binomial Probability

"Exactly" - ____

"At Most" - ____

"At Least" - ____

n= ____________________

p = ____

r = ____

Example1: "Exactly" Infection

8 out of 10 people will recover. If a group of 7 people become infected, what is the probability that exactly 3 people will recover?

Example 2: "At Most"

8 coins are tossed. What is the probability of getting at most 3 heads?

Example 3: "At Least"

For a certain species of mahogany tree, the survival rate is 90%. If 5 trees are planted, what are the probability that at least 2 trees die?