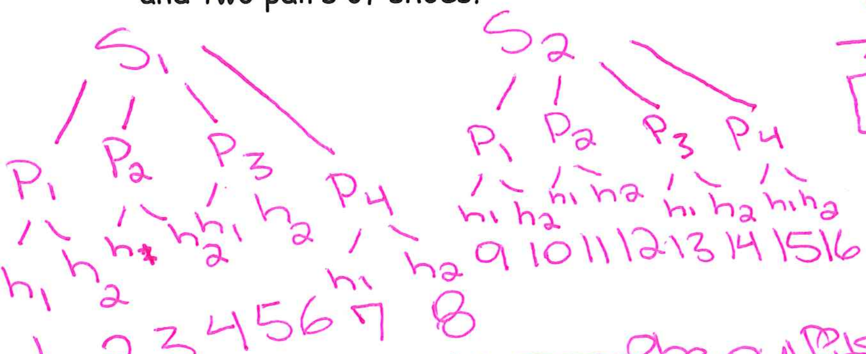
AFM Notes, Unit 1 – Probability   Name____________________

1-1 FPC and Permutations   Date____________ Period _____

<u>The Fundamental Principle of Counting</u>

How many different outfits could you put together using two sweaters, four pairs of pants, and two pairs of shoes?



16 possible outfits

Box method

S = sweaters
P = pants
h = shoes

$\underset{S}{\boxed{2}} \cdot \underset{P}{\boxed{4}} \cdot \underset{S}{\boxed{2}}$

↑ # of objects

The **Fundamental Principle of Counting** says: Suppose there are "a" ways of choosing one item, and "b" ways of choosing a second item, and "c" ways of choosing a third item, and so on. Then the total number of possible outcomes is $a \cdot b \cdot c$.

The probability of an event is: P(Event) = $\dfrac{\text{\# of ways "Event" can happen}}{\text{Total \# of possible outcomes}}$

Ex 1) Suppose a license plate can have any three letters followed by any four digits.

a) How many different license plates are possible?

$\boxed{26} \cdot \boxed{26} \cdot \boxed{26} \cdot \boxed{10} \cdot \boxed{10} \cdot \boxed{10} \cdot \boxed{10}$

letters   Numbers

175,760,000 possible plates

b) How many license plates are possible that have no repeated letters or digits?

$\boxed{26} \cdot \boxed{25} \cdot \boxed{24} \cdot \boxed{10} \cdot \boxed{9} \cdot \boxed{8} \cdot \boxed{7}$

78,624,000 possible plates without repetition

c) What is the **probability** that a randomly selected license plate has no repeated letters or digits?

without

P(plate w/o repitition) $= \dfrac{78{,}624{,}000}{175{,}760{,}000}$

$\dfrac{378}{845} \approx 0.447 \approx 44.7\%$

Permutations

Ex. 2) I have five books I want to arrange (in a particular order) on a shelf.

a) How many different ways can I arrange them?

$\boxed{5} \cdot \boxed{4} \cdot \boxed{3} \cdot \boxed{2} \cdot \boxed{1}$

# of books

120 ways

b) What if I only want to arrange 3 of my 5 books on a shelf? How many ways can I do this?

$\boxed{5} \cdot \boxed{4} \cdot \boxed{3}$ 60 ways

Whenever you want to know how many ways there are of arranging (in order) some number of items, that's called a permutation.

With **Permutations**
ORDER MATTERS

Ex 3: Seven flute players are performing in an ensemble.

a) The names of all seven players are listed in the program in random order. How many different ways could the players' names be listed (i.e., **arranged**) in the program?

$\boxed{7} \cdot \boxed{6} \cdot \boxed{5} \cdot \boxed{4} \cdot \boxed{3} \cdot \boxed{2} \cdot \boxed{1}$ 5040 ways

# of players

b) How many different ways could the players' names be listed in alphabetical order by last name?

1 way

c) If the players' names are listed in the program in random order, what is the **probability** that the names happen to be in alphabetical order?

$P(\text{alpha order}) = \frac{1}{5040} \approx 0.000198$

d) After the performance, the players are backstage. There is a bench with only room for four to sit. How many possible **arrangements** are there for four of the seven players to sit on the bench?

$\boxed{7} \cdot \boxed{6} \cdot \boxed{5} \cdot \boxed{4} = 840$ ways

# of seats

AFM Notes, Unit 1 – Probability
1-2 Combinations

Name________________________
Date______________ Period ________

Permutations vs. Combinations (Electing Officers vs. Forming a Committee)

Ex. 1) We want to elect three officers from our club of 25 people. The first person elected will be the President, the second person elected will be the Vice President, and the third person elected will be the Treasurer. How many different "arrangements" of officers can we have?

Permutations order matters $25P3 = 13,800$

P ← Permutation

n r

Total to select from

\# to be selected

math → PRB

nPr 253 =

Entur

Ex. 2) We want to form a 3-person committee (i.e., no officers) from our club of 25 people. How many committees can we form? Combination → order doesnt matter

$_{25}C_3 = 2,300$

n C r

total # of option

\# being selected

$\boxed{25 \cdot 24 \cdot 23}$ 1·1·23

$\boxed{1} \cdot \boxed{24} \cdot \boxed{23}$

President

When you're counting how many ways there are to arrange some number of items, order matters; that's a permutation.

When you're counting how many ways there are to simply group some number of items, order does not matter; that's a combination.

Ex. 3) The Debate Club wants to elect four officers (Pres, VP, Sec, and Treas), from its membership of 30 people. How many different ways could the Debate Club elect its officers?

Permutation $_{30}P_4 = 657,720$ ways

Ex. 4) The Debate Club wants to create a 4-person committee (i.e., no officers) from its membership of 30 people. How many different committees are possible?

Combination → order doesnt matter

$_{30}C_4 = 27,405$ ways

Combinations with Restrictions

Ex. 5) The Young Republicans Club consists of 7 seniors, 9 juniors, and 5 sophomores. They want to form a Planning Committee (i.e., without officers) to plan their spring social. The Planning Committee will consist of 4 members. = 21

a) How many different 4-member committees are possible? Combination

$_{21}C_4 = 5,985$

b) How many committees are possible that consist of all sophomores? Restriction

Sophomores only

$_5C_4 = 5$

c) How many different committees could be formed if the club's president must be one of the members?

Restriction:

1 member must be president

President: $\boxed{1}$

Committee: $\underbrace{\boxed{}\ \boxed{}\ \boxed{}}_{_{20}C_3 = 1,140}$

d) How many different committees could be formed if the committee must contain exactly two seniors and two juniors?

Restrictions:

2 seniors

2 juniors

$S_1 = 36$

$S_2 = 36$

Seniors: $\boxed{} \cdot \boxed{}$ $\cdot$ Juniors: $\boxed{} \cdot \boxed{}$

$_7C_2 = 21$ $\quad$ $_9C_2 = 36$

$21 \cdot 36 = 756$

AFM Notes, Unit 1 – Probability
1-3 Factorial Notation

Name_______________
Date__________ Period ______

Ex. 1) I have eight books I want to arrange on a shelf.

a) How many different ways can I arrange the eight books?

1) Using the permutations operation on the calculator $_8P_8 = 40{,}320$
2) Using the Fundamental Principle of Counting $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$

A third way to express this answer is by using factorial notation:

$$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$$

b) What if I only want to arrange 3 of my 8 books on a shelf? How many ways can I do this?

Again, we've already discussed two ways to calculate the answer to this problem.

1) Using the permutations operation on the calculator $_8P_3 = 336$
2) Using the Fundamental Principle of Counting $8 \cdot 7 \cdot 6 = 336$

We can also express this answer by using factorial notation

$$\frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 336 \qquad {}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$$

This last expression is actually the formula for a permutation. If we want to calculate the number of permutations of n objects taken r at a time, we would write:

$$_nP_r = \frac{n!}{(n-r)!}$$

Ex. 2) Calculate the expression 120!/116!

$$\frac{120 \cdot 119 \cdot 118 \cdot 117 \cdot \cancel{116} \cdot \cancel{115} \cdot \cancel{114} \cdot \cancel{113}}{\cancel{116} \cdot \cancel{115} \cdot \cancel{114} \cdot \cancel{113}} = 197{,}149{,}680$$

Ex. 3) Calculate the expression 76!/73!

$$\frac{76\cdot75\cdot74\cdot\cancel{73}\cdot\cancel{72}\cdot\cancel{71}\cdot\cancel{70}}{\cancel{73}\cdot\cancel{72}\cdot\cancel{71}\cdot\cancel{70}} = 421{,}800$$

Ex. 4) Calculate the expression n!/(n-3)!

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\cancel{(n-3)}\cancel{(n-4)}\cancel{(n-5)}}{\cancel{(n-3)}\cancel{(n-4)}\cancel{(n-5)}}$$

$$= n(n-1)(n-2)$$

$$= n^3 - 3n^2 + 2n$$

Ex. 5) Calculate the expression $_nP_{n-3}$

$$_nP_{n-3} = \frac{n!}{(n-r)!} = \frac{n!}{(\cancel{n}-\cancel{(n}-3))!}$$

$$= \frac{n!}{3!} = \frac{n!}{3\cdot2\cdot1} = \frac{n!}{6}$$

Math

→ PRB

#4

AFM Notes, Unit 1 - Probability

1-4 Introduction to Probability

Name ______________________

Date ____________ Period ____________

Probability theory was initially developed in 1654 in a series of letters between two French mathematicians, Blaise Pascal and Pierre de Fermat, as a means of determining the fairness of games. It is still used today to make sure that casino customers lose more money than they win, and in many other areas, including setting insurance rates.

At the heart of probability theory is randomness. Rolling a die, flipping a coin, drawing a card and spinning a game board spinner are all examples of random process. In a random process no individual event is predictable, even though the long range pattern of many individual events often is predictable.

Types of Probability

Experimental - The probabilities based on real-world data

Theoretical - The probabilities not based on real-world data.

Calculating Probabilities

When calculating the probability of something happening, the "something" is called an event, and the probability of the event happening is written P(event).

Ex. 1a) The probability of rolling a 3 on a die would be written P(rolling a 3).

Ex. 1b) The probability of winning the lottery would be written P(winning the lottery)

Probabilities are always expressed as a number between 1 + Ø. The probability of an event that is certain to happen is 1, while the probability of an impossible event is 0.

To calculate a probability, you count the # of ways an event can happen and divide this number by the total # of possible outcomes.

Probability of an event: P(E) =

$$\frac{\text{\# of ways an event can happen}}{\text{\# of possible outcomes}}$$

Example of Theoretical Probability

Ex. 2) A bag contains 4 blue marbles, 6 green marbles and 3 yellow marbles. A marble is drawn at random from the bag.

a) What's the probability of drawing a green marble?

$P(\text{Green}) = \frac{6}{13} = 0.462 = 46.2\%$

b) What's the probability of drawing a yellow marble?

$P(\text{yellow}) = \frac{3}{13} = 0.231 = 23.1\%$

c) What's the probability of drawing a green OR yellow marble?

$P(\text{green or yellow}) = \frac{6}{13} + \frac{3}{13} = \frac{6+3}{13} = \frac{9}{13} = 0.692 = 69.2\%$

Addition Rule

Example of Experimental Probability

Ex. 3) Suppose a study of car accidents and drivers who use mobile phones produced the following data:

Contingency Table	Had a car accident in the last year	Did not have a car accident in the last year	Totals
Driver using mobile phone	45	280	325
Driver not using mobile phone	25	405	430
Totals	70	685	755

This type of table is called a Frequency (Contingency) Table

The total number of people in the sample is 755. The row totals are 325 and 430. The column totals are 70 and 685. Notice that 325 + 430 = 755, and 70 + 685 = 755.

Calculate the following probabilities using the table above:

a) P(a driver is a mobile phone user) = $\frac{325}{755} = 0.430 = 43\%$

b) P(a driver had no accident in the last year) = $\frac{685}{755} = 0.907$ 90.7%

c) P(a driver using a mobile phone had no accident in the last year) =

$= \frac{280}{325} = 0.862 = 86.2\%$

AFM Notes, Unit 1 Probability
1-5 More Probability

Name ______________________
Date ____________ Period ____________

A compound probability is a probability involving two or more events, for example, the probability of Event A AND Event B happening.

Example 1: Coin Flip
What's the probability of flipping a coin twice and having it come up heads both times?

P(Comes up heads) = $\frac{\text{\# of ways an "Event" can happen}}{\text{Total \# of possible outcomes}}$

P(1st flip comes up heads AND 2nd flip comes up heads)

$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25 = 25\%$

The Multiplication Rule
When calculating the probability of two events, Event A and Event B, if the events are dependent, then the probability of both events happening is $P(A) \cdot P(B)$

Compound Probability and Replacement
Example 2: Marbles
A bag contains 4 blue marbles, 6 green marbles and 3 yellow marbles. If two marbles are drawn at random from the bag, what's the probability of:

a) First drawing a green marble, and then drawing a yellow marble?

With replacement

P(Green AND Yellow) =

$= \frac{6}{13} \cdot \frac{3}{13} = \frac{18}{169} = 0.107$

10.7%

Without replacement

P(Green AND Yellow)

$= \frac{6}{13} \cdot \frac{3}{12} = \frac{18}{156}$

$= 0.115 = 11.5\%$

b) Drawing two blue marbles?

With replacement

P(Blue And Blue)

$= \frac{4}{13} \cdot \frac{4}{13} = \frac{16}{169} = 0.095$

9.5%

Without replacement

P(Blue AND Blue)

$= \frac{4}{13} \cdot \frac{3}{12} = \frac{1}{13} = 0.077$

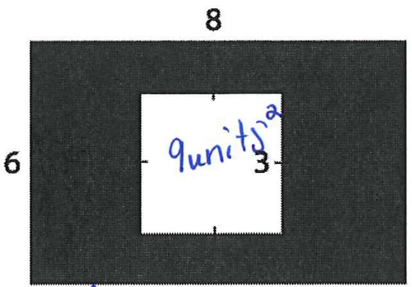
7.7%

Geometric Probability - a probability that is found by calculating a ratio of lengths or area of a geometric figure.

P(Event) = $\frac{\text{\# of ways an "Event" can happen}}{\text{Total \# of possible outcomes}} = \frac{\text{Event Area}}{\text{Total Area}}$

Example 3: Geometric Probability of Rectangles

a) What is the probability that a point chosen at random in the rectangle will also be in the square.



A=lw

$P(\text{point in square}) = \frac{\text{Event Area}}{\text{Total Area}}$

$= \frac{\text{Area of Square}}{\text{Area of Rectangle}}$

$= \frac{9}{48} = 0.188 = 18.8\%$

(b) What is the probability that a point chosen at random in the rectangle will be in the shaded area?

$P(\text{point in shaded region}) = \frac{\text{Event Area}}{\text{Total Area}} = \frac{\text{Area of Rectangle} - \text{Area of Square}}{\text{Area of Rectangle}}$

$= \frac{48-9}{48} = \frac{39}{48} = \frac{13}{16} = 0.813 = 81.3\%$

Example 4: Geometric Probability of Circles

The radius of the inner circle is 6cm, and the radius of the outer circle is 16cm. Find the probability that a point selected at random in the outer circle will be in the

$A = \pi r^2$

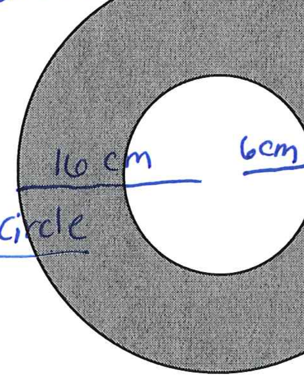
$A = \pi 16^2 \approx 804.248$

$A = \pi 6^2 \approx 113.097$

(a) inner circle

$P(\text{inner circle}) = \frac{113.097}{804.248} = 0.141$

14.1%



(b) shaded area

$P(\text{outer circle}) = \frac{\text{Area of outer Circle} - \text{Area Inner Circle}}{\text{Area of outer Circle}}$

$= \frac{804.248 - 113.097}{804.248}$

$= \frac{691.151}{804.248} = 0.859$

85.9%

Mutually Exclusive Events

When you roll a die, an event such as rolling a 1 is called a Simple event. because it consists of only one event.

An event that consists of two or more simple events is called a compound event. Such as the event of rolling an odd number or a number greater than 5.

Mutually exclusive events is when two evens cannot occur at the same time. Like the probability of drawing a 2 or an ace is found by adding their individual probabilities.

If two events, A and B, are mutually exclusive, then the probability of A or B occurs is the sum of their probabilities.

P(A or B) = P(A) + P(B)

Example 5: Two Mutually Exclusive Events
Keisha ha a stack of 8 baseball cards, 5 basketball cards, and 6 soccer cards. If she selects a card at random from a stack, what is the probability that it is a baseball or a soccer card?

P(baseball or soccer) = P(baseball) + P(soccer)

$$= \frac{8}{19} + \frac{6}{19} = \frac{14}{19}$$

The probability that Keisha selects baseball or soccer card is about 74%.

Example 6: Three Mutually Exclusive Events
There are 7 girls and 6 boys on the junior class homecoming committee. A subcommittee of 4 people is being chosen at random to decide the theme for the class float. What is the probability that the subcommittee will have a least 2 girls?

$$\frac{C(7,2)\cdot C(6,2)}{C(13,4)} + \frac{C(7,3)\cdot C(6,1)}{C(13,4)} + \frac{C(7,4)\cdot C(6,0)}{C(13,4)}$$

$$= \frac{315}{715} + \frac{210}{715} + \frac{35}{715} = \frac{112}{143} = 78\%$$

Inclusive Events

Since it is possible to draw a card that is both queen and a diamond, these events are not mutually exclusive, they are inclusive events.

If two events, A and B, are inclusive, then the probability that A or N occurs is the sum of their probabilities decreased by the probability of both occurring.

P(A or B) = P(A) + P(B) - P(A and B)

Example 7: Education
The enrollment at Southburg High school is 1400. Suppose 550 students take French, 700 take algebra, and 400 take both French and algebra. What is the probability that a student selected at random takes French or algebra?

$$\frac{550}{1400} + \frac{700}{1400} - \frac{400}{1400} = \frac{17}{28} = 61\%$$

Conditional Probability

The probability of an event under the condition that some preceding even has occurred is called conditional probability. The conditional probability that event A occurs given that event B occurs can be represented by P(A | B).

The conditional probability of event A, given event B, is defined as

Example 8: Medicine
Refer to the application below. What is the probability that a test subject's hair grew, given that he used the experimental drug?

	Number of Subjects	
	Using Drug	Using Placebo
Hair Growth	1600	1200
No Hair Growth	800	400

$$\frac{\frac{1600}{4000}}{\frac{2800}{4000}} = \frac{1600}{2400} = \frac{2}{3} = 67\%$$

AFM Notes, Unit 1 Probability

1-6 Probability Distribution

Name ______________________

Date __________ Period __________

Probability Distribution can be a table, graph, or equation that links each possible outcome of an event with its probability of occurring.

- The probability of each outcome must be between 0 and 1.
- The sum of all the probabilities must equal 1.

Making a Probability Distribution

Example 1: Bakery

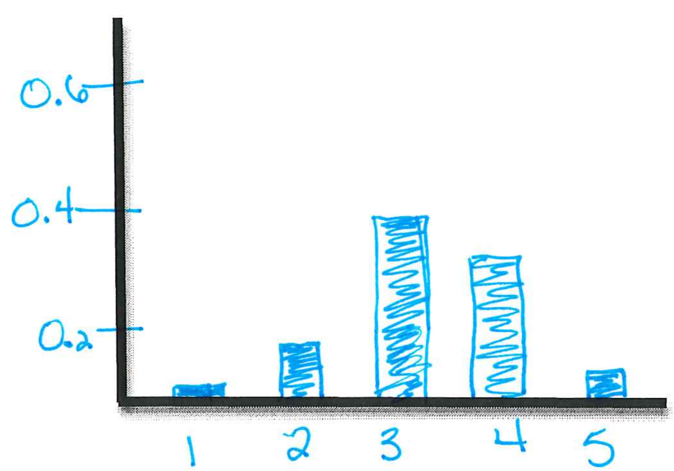
A bakery is trying a new recipe for the fudge deluxe cake. Customers were asked to rate the flavor of the cake on a scale of 1 to 5, with 1 being not tasty, 3 being okay, and 5 being delicious. Use the frequency distribution show to construct and graph a probability distribution.

Step 1: Find the proabability of each score.

Score, x	Frequency	P(x)
1	1	1/50 = .02
2	8	8/50 = .16
3	20	20/50 = .40
4	16	16/50 = .32
5	5	5/50 = .10

50

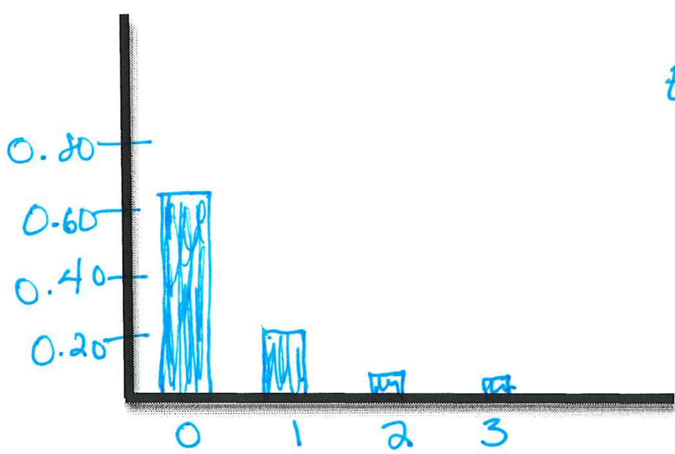
Step 2: Graph the score versus the probability.



Example 2: Car Sales

A car salesperson tracked the number of cars she sold each day during a 30-day period. Use the frequency distribution of the results to construct and graph a probability didstribution for the random variable x, rounding each proability to the nearest hundreth.

Cars Sold, x	Frequency	P(x)
0	20	$20/30 = 0.667$
1	7	$7/30 = 0.233$
2	2	$2/30 = 0.067$
3	1	$1/30 = 0.033$



$EV = p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4$

Expected Value

$= (.667 * 0) + (.233 * 1) + (.067 * 2) + (0.033 * 3)$

$= 0 + .233 + .0335 + .099$

$= 0.466$

Expected Value

In a random experiment, the values of the n outcomes are $x_1, x_2, x_3, \cdots x_n$ and the corresponding probabilities of the outcomes occurring are $p_1, p_2, p_3, \cdots p_n$.

The expected value (EV) of the experiment is given by:

$EV = p_1 x_1 + p_2 x_2 + p_3 x_3 + \cdots + p_n x_n$

To calculate expected value:

- Start with the probability Distribution or create it if you don't have it.
- Multiply the value of each outcome by it's probability.
- Add up all those products.
- The sum is the expected value.

Example 3: Fundraisers

At a raffle, 500 tickets are sold at $1 each for three prizes of $100, $50, and $10. What is the expected value of your net gain if you buy a ticket?

Gain, X	$100 - $1 or $99	$50 - $1 or $49	$10 - $1 or $9	$0 - 1 or -$1
Probability P(x)	$\frac{1}{500} = 0.002$	$\frac{1}{500} = 0.002$	$\frac{1}{500} = 0.002$	$\frac{497}{500} = 0.994$

$EV = (0.002*99) + (0.002*49) + (0.002*9) + (0.994*(-1)) = -\$0.68$

Example 4: Water Park

A water park makes $350,000 when the weather is normal and loses $80,000 per season when there are more bad weather days than normal. If the probability of having more bad weather days than normal this season is 35%, find the park's expected profit.

Gain, x	350,000	-80,000
Probability, P(x)	.65	.35

$EV = (.65 * 350{,}000) + (.35 * -80{,}000)$

$= \$199{,}500$

Example 5: MP3 Players

Construct a probability distribution and find the expected value:

Students were asked how many MP3 players they own.

Players, x	Frequency	P(x)
0	9	0.214
1	17	0.405
2	9	0.214
3	5	0.119
4	2	0.048

$EV = (.214*0) + (.405*1) + (.214*2) + (.119*3) + (0.048*4)$

$= 1.382$

Calculator Stat, Enter, Put values in L1, L2, Stat, calc, 2varstat

Down

Σxy

AFM Notes, Unit 1 Probability

1-7 Binomial Theorem

Name Key

Date ____________ Period ____________

According to the U.S. Census Bureau, ten percent of families have three or more children. If a family has four children, there are six sequences of births of boys and girls that result in two boys and two girls. Find these sequences.

BBGG BGBG BGGB GBBG GBGB GGBB

Pascal's Triangle

You can use the coefficients in powers of binomials to count the number of possible sequences in situation such as the one above. Remember that a binomial is a polynomial with two terms. Expand a few powers of the binomial b + g.

$(b+g)^0 = 1b^0g^0$

$(b+g)^1 = 1b^1g^0 + 1b^0g^1$

$(b+g)^2 = 1b^2g^0 + 2b^1g^1 + 1b^0g^1$

$(b+g)^3 = 1b^3g^0 + 3b^2g^1 + 3b^1g^2 + 1b^0g^3$

$(b+g)^4 = 1b^4g^0 + 4b^3g^1 + 6b^2g^2 + 4b^1g^3 + 1b^0g^4$

The coefficient 6 of the b^2g^2 in the expansion of $(b + g)^4$ gives the number of sequences of births that result in two boys and two girls. As another example the coefficient 4 of the b^1g^3 term gives the number of sequences with one boy and three girls.

Here are some patterns that can be seen in any binomial expansion of the form $(a + b)^n$.

1. There are $n+1$ terms.
2. The exponent n of $(a+b)^n$ is the exponent of *a* in the first term and the exponent of *b* in the last term.
3. In successive terms, the exponent of *a* decreases by one, and the exponent of *b* increases by one.
4. The sum of the exponents in each term is n.
5. The coefficients are symmetric. They increase at the beginning of the expansion and decrease at the end.

Notes 1-7 Binomial Theorem

The coefficients form a pattern that is often displayed in a triangular formation. This is known as Pascal's Triangle. Notice that each row begins and ends with 1. Each coefficient is the sum of the two coefficients about t in the previous row

$(a+b)^0$						1						
$(a+b)^1$					1		1					
$(a+b)^2$				1		2		1				
$(a+b)^3$			1		3		3		1			
$(a+b)^4$		1		4		6		4		1		
$(a+b)^5$	1		5		10		10		5		1	

Example 1: Use Pascal's Triangle
Expand $(x+y)^7$
Write two more rows of Pascal's triangle.

1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Use the patterns of a binomial expression and the coefficients to write the expansion of $(x+y)^7$

$(x+y)^7 = 1x^7y^0 + 7x^6y^1 + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7x^1y^6 + 1x^0y^7$

$= x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

The Binomial Theorem

Another way to show the coefficients in a binomial expansion is to write them in terms of the previous coefficients.

$(a+b)^0$	1				
$(a+b)^1$	1	$\frac{1}{1}$			
$(a+b)^2$	1	$\frac{2}{1}$	$\frac{2\cdot 1}{1\cdot 2}$		
$(a+b)^3$	1	$\frac{3}{1}$	$\frac{3\cdot 2}{1\cdot 2}$	$\frac{3\cdot 2\cdot 1}{1\cdot 2\cdot 3}$	
$(a+b)^4$	1	$\frac{4}{1}$	$\frac{4\cdot 2}{1\cdot 2}$	$\frac{4\cdot 3\cdot 2}{1\cdot 2\cdot 3}$	$\frac{4\cdot 3\cdot 2\cdot 1}{1*2*3*4}$

This pattern provides the coefficients of $(a+b)^n$ for any nonnegative integer n. The pattern is summarized in the Binomial Theorem.

If n is a nonnegative integer, then

$$(a+b)^n = 1a^n b^0 + \frac{n}{1}a^{n-1}b^1 + \frac{n(n-1)}{1\cdot 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}a^{n-3}b^3 + \ldots + 1a^0b^n$$

Example 2: Use the Binomial Theorem

Expand $(a-b)^6$

The expansion will have seven terms. Use the sequence $1\ \frac{6}{1}\ \frac{6}{1}\cdot\frac{5}{2}\ \frac{6\cdot 5\cdot 4}{1\cdot 2\cdot 3}$ to find the coefficients for the first four terms. Then use symmetry to find the remaining coefficients.

$$(a-b)^6 = 1a^6(-b)^0 + \frac{6}{1}a^5(-b)^1 + \frac{6\cdot 5}{1\cdot 2}a^4(-b)^2 + \frac{6\cdot 5\cdot 4}{1\cdot 2\cdot 3}a^3(-b)^3 + \ldots + 1a^0(-b)^6$$

$$= a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

That the terms in the middle have the same coefficients, the exponents are reversed

Note:

, as in $15a^4b^2$ and $15a^2b^4$.

The factors in the coefficients of binomial expansion involve special products called factorials. For example, the product of $4 \cdot 3 \cdot 2 \cdot 1$ is written 4! And is read as 4 factorial. In general, if n is positive integer, then $n! = n(n-1)(n-2)(n-3)\ldots 2 \cdot 1 \quad 0! = 1$.

Example 3: Factorials

Evaluate $\frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{3! \cancel{5!}} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = \frac{336}{6} = 56$

An expression such as $\frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3}$ in Example 2 can be written as a quotient of factorials. In this case $\frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3} = \frac{6!}{3!3!}$. Using this idea, you can rewrite the expansion of $(a + b)^6$ using factorials:

$$(a+b)^6 = \frac{6!}{6!0!}a^6b^0 + \frac{6!}{5!1!}a^5b^1 + \frac{6!}{3!3!}a^3b^3 + \frac{6!}{2!4!}a^2b^4 + \frac{6!}{1!5!}a^1b^5 + \frac{6!}{0!6!}a^0b^6$$

You can also write this series using sigma notation.

$$(a+b)^6 = \sum_{k=0}^{6} \frac{6!}{(6-k)!\,k!} \cdot a^{6-k}b^k$$

In general, the Binomial Theorem can be written both in factorial notation and in sigma notation.

Example 4: Use a Factorial Form of the Binomial Theorem

$$(2x+y)^5 = \sum_{k=0}^{5} \frac{5!}{(5-k)!\,k!} 2x^{5-k} y^k$$

$$= \frac{5!}{5!\,0!}(2x)^5 y^0 + \frac{5!}{4!\,1!}(2x)^4 y^1 + \frac{5!}{3!\,2!}(2x)^3 y^2 + \frac{5!}{2!\,3!}(2x)^2 y^3 + \frac{5!}{1!\,4!}(2x)^1 y^4 + \frac{5!}{0!\,5!}(2x)^5 y$$

$$= 32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$$

Sometimes you need to know only a particular term of a binomial expansion. Note that when the Binomial Theorem is written in sigma notation, k = 0 for the first time, k = 1 for the second term, and so on. In general, the value of k is always one less than the number of term you are finding.

Example 5: Find a Particular Term

Find the fifth term in the expansion of $(p + q)^{10}$.

First, use the Binomial Theorem to write the expansion in sigma notation.

$$(p+q)^{10} = \sum_{k=0}^{10} \frac{10!}{(10-k)!\,k!} p^{10-k} q^k$$

If the fifth term, k = 4.

$$\frac{10!}{(10-k)!\,k!} p^{10-k} q^k = \frac{10!}{(10-4)!\,(4)!} p^{10-4} q^4$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} p^6 q^4$$

$$= 210\,p^6 q^4$$

AFM Notes, Unit 1 Probability

1-8 Binomial Probability

Name Key

Date ____________ Period ____________

Conditions of a Binomial Experiment:

A binomial experiment exists if and only if these conditions occur.

- Each trial has exactly two outcomes, or outcomes that can be reduced to two outcomes.
- There must be a fixed number of trials.
- The out comes of the trial must be independent.
- The probabilities in each trial are the same.

Binomial Probability

"Exactly" - binompdf(n, p, r)

"At Most" - binomcdf(n, p, r)

"At Least" - 1 - binomcdf(n, p, r-1)

n= # of trials

p = probability of success in each trial

r = how many successes you expect or predict over all trials

Example1: "Exactly" Infection

8 out of 10 people will recover. If a group of 7 people become infected, what is the probability that exactly 3 people will recover?

binompdf(n, p, r)

$n=7$
$p=0.8$
$r=3$

2nd vars
A) binompdf(n, p, r)
enter

binompdf(7, 0.8. 3) = 0.0287
2.87%

Example 2: "At Most"

8 coins are tossed. What is the probability of getting at most 3 heads?

$n=8$
$p=0.5$
$r=3$

binomcdf(8, 0.5, 3) = .36
36%

Example 3: "At Least"

For a certain species of mahogany tree, the survival rate is 90%. If 5 trees are planted, what are the probability that at least 2 trees die?

$n=5$
$p=0.1$
$r=2-1$

1 - binomcdf(5, 0.1, 2-1) = 0.08146
8.146%